Forced Oscillations of the Vertical Rack

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Keywords: Vertical racks, Transmission line supports, Wind power plants, Advertising and signage media, Lighthouses for water and air navigation, Transverse vibrations of a vertical rack of constant cross-section, Transparency of algorithms and computer programs, The bending stiffness of the rod, Coefficient of linear-viscous friction, The method of separating variables.

Abstract: Vertical racks are frequently used in construction as transmission line supports, chimneys, searchlights, and antenna devices. They provide terrestrial and satellite television, amateur and professional television and radio communications, street and square lighting, telemetry and remote control systems, wind power installations, meteorological instruments and equipment, advertising and signage carriers, and lighthouses for water and air navigation. Vertical racks are extensively used. Analytical approaches are typically inefficient in tackling difficult issues in oscillation theory and its applications in engineering. The technical theory of beam bending vibrations is applied.

1 INTRODUCTION

Vertical racks have a common use in construction as supports for transmission lines, chimneys, searchlights, antenna devices, and so on. They provide terrestrial and satellite television, amateur and professional television and radio communications, street and square lighting, telemetry and remote control systems, wind power installations, meteorological instruments and equipment, advertising and signage carriers, lighthouses for water and air navigation, and so on. The widespread usage of vertical racks has sparked substantial interest in their oscillations due to different disturbances (Kulterbaev, Abdul Salam, 2018; Kulterbaev, 2011). The transverse vibrations of a vertical rack with a constant cross-section are examined in this study under the influence of dynamic harmonic disturbances. Analytical approaches are typically inefficient in tackling difficult issues in oscillation theory and its applications in engineering. For elastic elements with simple geometry, the finite difference method is very attractive, characterized by simplicity and transparency of algorithms and computer programs.

2 PROBLEM STATEMENT

Under the action of a transverse uniformly distributed load, a homogeneous vertical rod (rack) of constant cross-section (Fig. 1) of length l with a linear mass m and an axial moment of inertia of the cross-section made of a material with an elastic modulus E performs bending vibrations described by the function u(x,t).

Let's use the technical theory of bending vibrations of beams. Then the procedure for deriving the equation of oscillations gives

$$bu^{IV} + m\ddot{u} + \eta m\dot{u} + p(u' + xu'') = q(t). \quad (1)$$

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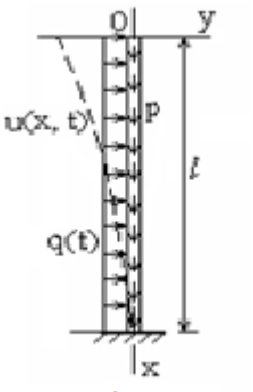

Figure 1

Figure 1: Homogeneous vertical rod (rack) of constant cross-section.

The notation is introduced here: $b = EJ$ - bending stiffness of the rod, η - coefficient of linear-viscous friction. The dot above the variables means the time derivative, the Roman numeral and the strokes in the superscript correspond to the x derivatives.

3 PROBLEM SOLUTION

Oscillations in steady-state with harmonic disturbance represented by a transverse distributed load (dynamic perturbation) $q(t)$. After transformation (1), the related mathematical model of the problem takes the form

$$u^{IV} + \theta(u' + xu'') + \gamma\dot{u} + \varepsilon\dot{u} = f(t), \quad x \in (0,\ l), \quad t > -\infty. \tag{2}$$

$$u''(0,t) = 0, \quad u'''(0,t) = 0, \quad u(l,t) = 0, \quad u'(l,t) = 0. \tag{3}$$

The notation is introduced here

$$\theta = p/b, \quad \gamma = m/b, \quad \varepsilon = \eta m/b,$$
$$f(t) = q(t)/b. \tag{4}$$

We take the dynamic load in the form of a complex - valued function

$$q(t) = a_q e^{j(\Omega t + \psi)} = A_q e^{j\psi t}, \qquad A_q = a_q e^{j\psi},$$

where a_q – actual amplitude, A_q – complex amplitude, ψ – initial phase. By virtue of (4)

$$f(t) = \frac{a_q}{EJ} e^{j(\Omega t + \psi)} = A_f e^{j\Omega t}, \qquad A_f = \frac{A_q}{EJ}.$$

The solution of the problem (2), (3) is sought using the method of separating variables as a product

$$u(x,t) = X(x)e^{\lambda t}, \tag{5}$$

where

$$\lambda = j\Omega \tag{6}$$

- characteristic index, X(x) is a function of the amplitude of forced oscillations to be determined. Substitute (5) in (2), (3) and we get

$$X^{IV} + \theta(X' + xX'') + \Lambda X = A_1, \quad \Lambda = \gamma\lambda^2 + \varepsilon\lambda, \quad x \in (0,\ l), \tag{7}$$

$$X''(0) = 0, \quad X'''(0) = 0, \qquad X(l) = 0, \qquad X'(l) = 0. \tag{8}$$

From the differential equation (7) we proceed to the finite difference scheme (Verzhbitsky, 2002; Amosov, Sinitsyn, 2001; Kulterbaev, 2003). It is not difficult to show that instead of (7) a finite difference equation will be obtained

$$y_{i-2} + c_i y_{i-1} + d_i y_i + e_i\ y_{i+1} + y_{i+2} = A_h, \qquad i = 3,\ 4,\ ...,\ n-2, \tag{9}$$

where indicated

$$c_i = -4 - 0{,}5\theta h^3 + \theta h^2 x_i, \qquad d_i = 6 - 2\theta h^2 x_i + h^4\Lambda,$$
$$e_i = -4 + 0{,}5\theta h^3 + \theta h^2 x_i, \qquad A_h = h^4 A_f.$$

We will bring the boundary conditions (8) in accordance with the finite difference method (Kulterbaev, Abdulsalam, 2018; Babakov, 1968) and write (Fig. 2):

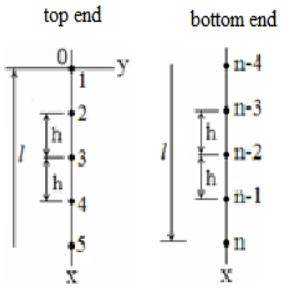

Figure 2

Figure 2: Bring the boundary conditions in accordance with the finite difference method.

At the upper end:

$$\begin{aligned} 2y_1 - 5y_2 + 4y_3 - y_4 &= 0, \\ -5y_1 + 18y_2 - 24y_3 + 14y_4 - 3y_5 &= 0. \end{aligned} \tag{10}$$

At the lower end:

$$y_n = 0, \qquad y_{n-2} - 4y_{n-1} + 3y_n = 0. \tag{11}$$

Equations (9) - (11) form an inhomogeneous algebraic system (Amosov, Sinitsyn, 2001), which can be rewritten in matrix-vector form

$$\mathbf{G}(\lambda)\,\mathbf{Y} = \mathbf{Q}, \qquad (12)$$

where $\mathbf{G}(\lambda)$ - a square matrix of order n. It can be easily shown that equation (12) in coordinate form has the form

$$\begin{pmatrix} 2 & -5 & 4 & -1 & \vdots & & & & & & \\ -5 & 18 & -24 & 14 & -3 & \vdots & & & & & \\ 1 & c_3 & d_3 & e_3 & 1 & \vdots & & & & & \\ & 1 & c_4 & d_4 & e_4 & \vdots & & & & & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & & & & & \vdots & 1 & c_{n-2} & d_{n-2} & e_{n-2} & 1 \\ & & & & & \vdots & & & 1 & -4 & 3 \\ & & & & & \vdots & & & & & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ \dots \\ y_{n-2} \\ y_{n-1} \\ y_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ A_h \\ A_h \\ \dots \\ A_h \\ 0 \\ 0 \end{pmatrix}. \qquad (13)$$

Here zero elements are not written out. The solution of the system of equations (12), (13) gives the vector

$$Y = G^{-1}Q,$$

and then the vector of oscillation amplitudes according to the formula

$$a_u(x_i) = |Y| \cdot$$

Example. Consider a steel rod of circular cross-section and a load with the following input data:

$$l = 3 \text{ м}, \quad n = 1001, \quad \eta = 0{,}5 \text{ с}^{-1},$$
$$d = 1 \text{ см}, \quad \rho = 7810 \text{ кг/м}^3,$$
$$E = 210 \text{ ГПа}, \quad J = \pi d^4/64,$$
$$\psi = 0, \quad a_q = 1{,}8 \text{ Н/м}.$$

According to the procedure described in (Kulterbaev, Abdulsalam, 2018; Biderman, 1979), three elements of the natural frequency spectrum are determined, presented in the table 1.

Table 1: Three elements of the natural frequency spectrum are determined.

k	1	2	3
ω_k, с$^{-1}$	4,52	31,32	88,54

Let's study the influence of increasing values of the frequency of disturbances

$$\Omega = \{\ 0 \quad 1{,}7 \quad 2{,}3 \quad 31 \quad 88{,}3\ \}\ \text{с}^{-1}$$

on the amplitude of the oscillations.

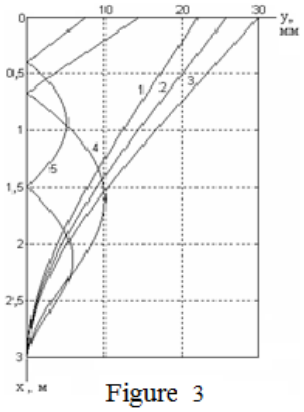

Figure 3

Figure 3: The curves in with numbers in the order of the vector Ω elements in the example condition.

The curves in Fig. 3, with numbers in the order of the vector Ω elements in the example condition, clearly show that as the frequency of disturbances increases, the amplitudes increase, and the shape of their distribution along the length of the rod coincides with the first eigenfunction. When the first eigenvalue coincides with the frequency of the perturbations, the oscillations become resonant and the amplitudes grow (the related graphs are not shown here). After the initial resonance, as the frequency of disturbances increases, vibrations occur according to the second eigenfunction (curve 4), and subsequently according to the third (curve 5), and so on.

Based on this analysis, it is possible to conclude that, when using forced oscillations of the vertical rack, the magnitude and forms of the amplitude distribution along the rack are significantly affected by the spectra of eigenvalues and functions of the generating homogeneous differential equation with homogeneous boundary conditions.

3 CONCLUSION

1. The magnitude and shapes of the amplitude distribution along the vertical rack with forced oscillations depend strongly on the spectra of the eigenvalues and functions of the generating homogeneous differential equation with homogeneous boundary conditions.

2. Using the approach of variable separation, the answer to the problem is simply determined.

3. The magnitude and shapes of the amplitude distribution along the vertical rack with forced oscillations depend strongly on the spectra of the

eigenvalues and functions of the generating homogeneous differential equation with homogeneous boundary conditions.

REFERENCES

Kulterbaev, H.P., Abdul Salam, I.M., 2018. Longitudinal free oscillations of a rod with discrete masses. Problems of mechanics and control: materials of the International Conference (September 16-22, 2018, Makhachkala). p. 2007-2010.

Kulterbaev, H.P., 2011. Kinematically excited oscillations of a continually discrete multi-span beam. *Bulletin of the Nizhny Novgorod University named after N.I. Lobachevsky. Proceedings of the X All-Russian Congress on Fundamental Problems of Theoretical and Applied Mechanics*, No. 4, part 2. pp. 198-200.

Verzhbitsky, V.M., 2002. Fundamentals of numerical methods. Moscow: Higher School. 840 p.

Kulterbaev, H.P., Abdulsalam, I.M., 2018. Longitudinal free vibrations of a rod with discrete masses. Problems of Mechanics and Control: Proceedings of the International Conference (September 16-22, 2018, Makhachkala). pp. 207-2010.

Amosov, A.A., Sinitsyn, S.B., 2001. Fundamentals of the theory of seismic resistance of structures. 96 p.

Babakov, I.M., 1968. Theory of oscillations. 560 p.

Biderman, V.L., 1979. Applied theory of mechanical vibrations. 416 p.

Kulterbaev, H.P., 2003. Fundamentals of the theory of oscillations. Fundamentals of theory, homework tasks, examples of solutions. Kabardino-Balkar State University. 130 p.