Algebraic Quasi – fractal Modeling of Complex Systems with Intermittent Links

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Keywords: Smart systems, algebraic quasi-fractal systems, algebraic quasi-fractal logic, digital transformation, measurement scales, financial security, financial and economic systems, forecasting, structural sustainability

Abstract: Paper contains brief summary of new instruments of description of complex systems based on the works on the theory of algebraic systems by A.I. Maltsev, on the works on of the theory of fractals by Benoit Mandelbrot, the works of A. Tarsky, and the monographs of Serdyukova N.A. and Serdyukov V.I.. A new instrument, algebraic quasi – fractal systems, for investigation of complex systems with intermittent connections are offered on this base.

1 INTRODUCTION. RESEARCH METHODOLOGY

The background for the processes of digital transformation in almost all spheres that ensure and link the functioning of social society were the mathematization of science noted in the work of I.G. Shafarevich (Shafarevich, 1999) and the development of technologies based on the latest achievements of natural sciences. In fact, modern digital transformation is the next stage of the introduction of mathematics into practice, which began with the advent of mathematics. In addition, it should be noted that digital transformation is also the next stage in the implementation of the systematic approach developed in 1937 by Ludwig von Bertalanfi. Algebra and logic are currently key areas in the study of digital transformation, and it can be considered as the most important mechanisms that ensure strategic, fundamental goals and achievements of digital transformation. The role of mathematical methodology and methods underlying the digitalization of complex intelligent systems is increasing. The paper considers, in particular, a brief survey of algebraic methods of digitalization of complex systems, namely, methods of algebraic quasi–fractal logic introduced and developed by the authors, and a new approach to description of systems with intermittent connections.

The concept of a quasi–fractal algebraic system with a countable number of quasi-fractal levels was introduced in (Serdyukova, 2018). The concept of a quasi–fractal algebraic system with the number of levels equal to ω_γ , where ω_γ is an ordinal, and the notion of a quasi – fractal operator QF were introduced in (Serdyukova, Serdyukov, 2021). Among the received practical applications were, for example, the following ones:

- issues of stability of smart systems, in particular, the concept of stability of quasi– fractal algebraic systems, as well as factor –flexible quasi-fractal systems, was introduced and investigated,
- tensor estimation of smart systems satisfying a certain specified property /P – property/ is constructed, (Serdyukova, 2018).

In (Serdyukova, Serdyukov, 2021; Serdyukova, Serdyukov, 2022), based on the concept of quasi – fractal Boolean algebra, an algebraic quasi – fractal logic of the first order is introduced and studied.

In (Serdyukova, Serdyukov, 2019), the connection of quasi – fractal algebraic systems with mathematical logic is considered, in particular, the function of digitalization of the algebra of propositions and the narrow predicate calculus

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(NPC) is constructed, which allows us to build numerical estimates of the truth values of closed formulas of the algebra of propositions and the narrow predicate calculus (NPC).

Analogs of theorems on the properties of a probability measure are obtained for the function of digitalization of the algebra of propositions and the narrow predicate calculus (NPC). As a practical application of the concept of algebraic quasi –fractal logic, an approach is proposed on the basis of which an algorithm for solving the problem of information reliability is developed. In (Serdyukova, Serdyukov, Kusminova, Shishkina, 2024) conditional groups' probability function and conditional digitalization function are introduced. Conditional groups' function of digitalization can be used to clarify the qualitative side of the forecast and its connection with the quantitative characteristics. The definition of a measurement error, connected with conditional digitalization function is given. As a practice application of this notion the explanation of possible causes of forecasting errors in smart systems /in smart university ranking systems, in financial - economic systems are got.

In (Serdyukova, 2018) an algorithm was developed for determining the number of synergistic effects of a system determined by a finite number of factors, and on this basis, measurement scales were constructed to capture the synergistic effects of the system. This allows one to significantly reduce the risks that arise during the operation of the system. In continuation of this, in (Serdyukova, Serdyukov, Kusminova, Kusnetsov, Shishkina, 2022) a risk analysis algorithm for smart planning systems and a risk analysis algorithm for smart control systems were built.

2 RESEARCH RESULTS. DISCUSSION OF THE RESULTS

2.1 Systems with Intermittent Connections Notation trough Bayesian Approach and a Conditional Digitalization Function

Using an algebraic quasi-fractal, one can write down systems with intermittent connections/ links. To do this, we apply a conditional digitalization function using the Bayesian approach, (Serdyukova, Serdyukov, 2021). The defining relations in a group define the connections, and so the Bayesian approach will work. The lack of connection at the upper level of the quasi-fractal weakens the connections at its lower levels, (Serdyukova, Serdyukov, 2021).

The breakdown of the connections of the system in the models of factors defining the system in the form of quasi -fractal groups is associated with the presence of free non-abelian subgroups in these models, i.e. with the amenability property. Breaking the links of the system S modeled by the group of factors

$\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ means that at least one of the defining relations, say $\{w_j(a_1, \dots, a_n) = 1$ falls out, then we get the following model for the system S in the form of a group $\boldsymbol{G_j} = \langle G_j = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I, i \neq j\rangle$. Then $\boldsymbol{G}$ is an epimorphic image of $\boldsymbol{G_j}$. So, $\boldsymbol{G} \cong \boldsymbol{G_j}/\boldsymbol{ker}\,\varphi_j$, here $\varphi_j : \boldsymbol{G_j} \to \boldsymbol{G}$; $Im\varphi_j = G$.

Systems with intermittent connections are useful in studying dynamics of complex systems. So, areas of application of algebraic quasi-fractal dynamics include the following ranges: dynamic planning, that is, decisions are made during system operation based on an analysis of the current situation.

Based on the work of S. McLane (Mac Lane, 1991), an approach that allows to use Category Theory to study temporal logic has been developed. Using this approach, the concept of P −purity, (Serdyukova, 1991), can be defined in the class of algebras of finite signature with equality and finite-ary operations, as well as in categories and in first-order logics. This technique and the fixed - point theorem make it possible to determine the level of sustainability of quasi-fractal logic of the first order.

So, let's define a quasi – fractal category.

Definition 1. A quasi – fractal category is a quasi – fractal monoid for the product in the general sense described in the following way.

A meta quasi – fractal graph $QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{A}^{\alpha}_k = (\langle A^{\alpha}_k; \Omega_k \rangle, \alpha \in \Lambda_k))$, here ω_γ is an ordinal, consists of objects $\boldsymbol{A}^{\alpha}_k = \mathcal{Ob}^{\alpha}_k = \{a^{\alpha}_k, b^{\alpha}_k, c^{\alpha}_k, \dots, | \alpha \in \Lambda_k\}$, |arrows

$\Omega_k = \mathcal{Ar}^{\alpha}_k = \{f^{\alpha}_k, g^{\alpha}_k, h^{\alpha}_k, \dots, | \alpha \in \Lambda_k\}$, and two operations run as follows:

Domain, which assigns to each arrow f^{α}_k an object $a^{\alpha}_k = dom f^{\alpha}_k$; codomain which assigns to each arrow f^{α}_k an object $b^{\alpha}_k = cod f^{\alpha}_k$. These operations on f^{α}_k are best indicated by displaying f^{α}_k as an actual arrow starting at its domain and ending at its codomain. A meta quasi – fractal category is a meta quasi – fractal graph with two additional operations: quasi – fractal identity, which assigns to each object a^{α}_kan arrow $id_{a^{\alpha}_k} = 1_{a^{\alpha}_k} : a^{\alpha}_k \to a^{\alpha}_k$; composition which assigns to each pair $\langle g_k^\alpha, f_k^\alpha \rangle$ of arrows with $domg_k^\alpha = codf_k^\alpha$ an arrow $g_k^\alpha \circ f_k^\alpha$ called their composite, with $g_k^\alpha \circ f_k^\alpha : domf_k^\alpha \to codg_k^\alpha$. This operation is pictured by the diagram

$$\begin{tikzcd} & b_k^\alpha \arrow[dr, "g_k^\alpha"] & \\ a_k^\alpha \arrow[ur, "f_k^\alpha"] \arrow[rr, "g_k^\alpha \circ f_k^\alpha"] & & c_k^\alpha \end{tikzcd}$$

These operations in a meta category are subjects to the two following axioms:

Associativity. For given objects and arrows in the configuration

$$a_k^\alpha \overset{f_k^\alpha}{\to} b_k^\alpha \overset{g_k^\alpha}{\to} c_k^\alpha \overset{m_k^\alpha}{\longrightarrow} d_k^\alpha$$

One always has the equality

$$m_k^\alpha \circ (g_k^\alpha \circ f_k^\alpha) = (m_k^\alpha \circ g_k^\alpha) \circ f_k^\alpha$$
(1.1)

This equation is represented pictorially by the statement that the following diagram is commutative:

$$\begin{tikzcd} a_k^\alpha \arrow[rr, "m_k^\alpha \circ (g_k^\alpha \circ f_k^\alpha) = (m_k^\alpha \circ g_k^\alpha) \circ f_k^\alpha"] \arrow[d, "f_k^\alpha"'] \arrow[drr, "g_k^\alpha \circ f_k^\alpha"'] & & d_k^\alpha \\ b_k^\alpha \arrow[rr, "g_k^\alpha"'] \arrow[urr, "m_k^\alpha \circ g_k^\alpha"] & & c_k^\alpha \arrow[u, "m_k^\alpha"'] \end{tikzcd}$$

Unit law. For all arrows$f_k^\alpha a_k^\alpha \to b_k^\alpha$ and $g_k^\alpha : b_k^\alpha \to c_k^\alpha$composition with the identity arrow $1_{b_k^\alpha}$ gives

$$1_{b_k^\alpha} \circ f_k^\alpha = f_k^\alpha \text{ and} g_k^\alpha \circ 1_{b_k^\alpha} = g_k^\alpha \qquad (1.2)$$

This axiom asserts that the identity arrow $1_{b_k^\alpha}$ of each object b_k^α acts as an identity for the operation of composition, whenever it makes sense. The arrows of any metacategory are often called its morphisms.

A quasi – fractal metacategory is to be any interpretation, which satisfies all these axioms.

Examples

1. An example is a quasi – fractal metacategory of quasi – fractal sets, which has objects all sets and quasi – fractal arrows all quasi – fractal functions, with the usual quasi – fractal identity functions and the usual composition of functions. Let's note, that one can consider a quasi – fractal set as a quasi – fractal algebraic system with empty signature, that is $QF_{k<\omega_\gamma}^\alpha(\boldsymbol{A}_k^\alpha = (\langle A_k^\alpha; \Omega_k = \emptyset \rangle, \alpha \in \Lambda_k))$, here ω_γ is an ordinal

$\boldsymbol{A}_k^\alpha = \langle A_k^\alpha; \Omega_k = \emptyset \rangle, \alpha \in \Lambda_k$ is an algebraic system of the level k, $QF_{k<\omega_\gamma}^\alpha(\boldsymbol{A}_k^\alpha = \langle A_k^\alpha; \Omega_k = \emptyset \rangle), \alpha \in \Lambda_k$ is a designation of a quasi-fractal system for the limit ordinal ω_γ, for the ordinal $\omega_\gamma + 1$ we have: $a_\alpha = \boldsymbol{A}_{\omega_\gamma+1}^\alpha = \langle A_{\omega_\gamma+1}^\alpha ; \Omega_{\omega_\gamma+1} = \emptyset \rangle$, $\alpha \in \Lambda_{\omega_\gamma}$ is an algebraic system of the level $\omega_\gamma + 1$, here ω_γ is the limit ordinal.

So, one can consider a quasi – fractal set as a quasi – fractal algebraic system with empty signature, that is $QF_{k<\omega_0}^\alpha(\boldsymbol{A}_k^\alpha = (A_k^\alpha, \alpha \in \Lambda_k)$ or as follows: $QF_{k<\omega_0}^\alpha(\boldsymbol{X}_k^\alpha, \alpha \in \Lambda_k)$ here ω_γ is an ordinal.

2. A quasi – fractal function means a quasi – fractal function with specified domain and specified codomain. A quasi – fractal function $QF_{k<\omega_\gamma}^\alpha(\boldsymbol{f}_k^\alpha : QF_{k<\omega_\gamma}^\alpha(\boldsymbol{X}_k^\alpha, \alpha \in \Lambda_k) \to QF_{k<\omega_\gamma}^\alpha(\boldsymbol{Y}_k^\alpha, \alpha \in \Lambda_k))$ consists of a quasi – fractal set $QF_{k<\omega_0}^\alpha(\boldsymbol{X}_k^\alpha, \alpha \in \Lambda_k)$, its domain, a quasi – fractal set $QF_{k<\omega_\gamma}^\alpha(\boldsymbol{Y}_k^\alpha, \alpha \in \Lambda_k)$ its codomain, and a rule, which assigns to each element $x_k^\alpha \in X_k^\alpha$ an element $f_k^\alpha x_k^\alpha \in Y_k^\alpha$, here ω_γ is an ordinal.

3. A quasi – fractal metacategory of all sets has as quasi – fractal object, all sets, as quasi – fractal arrows, all quasi – fractal functions with the usual composition. The quasi – fractal metacategory of all groups is described in the following way: objects are all quasi – fractal groups, arrows are all quasi – fractal homomorphisms of groups. The quasi – fractal arrows of any quasi – fractal metacategory are called

its quasi – fractal morphisms. Since the quasi – fractal objects of a quasi – fractal metacategory corresponds exactly to its quasi – fractal identity arrows, it is technically possible to dispense altogether with the quasi – fractal objects and deal only with quasi – fractal arrows. The data for quasi – fractal arrows - only quasi – fractal metacategory $QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{\mathcal{C}} = (A^{\alpha}_k = \mathcal{Ob}^{\alpha}_k, \Omega_k = \mathcal{Ar}^{\alpha}_k, \alpha \in \Lambda_k))$, here ω_γ is an ordinal, consist of quasi – fractal arrows, certain ordered quasi – fractal pairs $\langle g^{\alpha}_k, f^{\alpha}_k \rangle$ called the composable pairs of quasi – fractal arrows, and an operation assigning to each composable quasi – fractal pair $\langle g^{\alpha}_k, f^{\alpha}_k \rangle$ an arrow $g^{\alpha}_k \circ f^{\alpha}_k$ called their composite. With these data one defines an identity of $QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{\mathcal{C}} = (A^{\alpha}_k = \mathcal{Ob}^{\alpha}_k, \Omega_k = \mathcal{Ar}^{\alpha}_k, \alpha \in \Lambda_k))$, here ω_γ is an ordinal, to be a quasi – fractal arrow u^{α}_k such that $f^{\alpha}_k \circ u^{\alpha}_k = f^{\alpha}_k$ whenever the composite $f^{\alpha}_k \circ u^{\alpha}_k$ is defined and $u^{\alpha}_k \circ g^{\alpha}_k = g^{\alpha}_k$ whenever the composite $u^{\alpha}_k \circ g^{\alpha}_k$ is defined.

The concept of a quasi - fractal category allows us to transfer the Erdos- Renyi algorithm and the concept of a giant component to almost any quasi - fractal algebraic systems with varying degrees of error and regardless of the amenability of these systems. If the quasi – fractal system $\mathfrak{A}$ is not an amenable one, then one can apply the Erdos- Renyi algorithm to elementary theory of this quasi - fractal algebraic system $Th(\mathfrak{A})$, (Serdyukova, Serdyukov, 2021).

2.2 Risk Systems of the System *S* in accordance with Model *G*. Risk Models Descriptions Using Homological Algebra Method

An important role in the study of the functioning of complex systems belongs to the study of the internal risks of these systems. As a rule, the risk of functioning of a complex system is manifested in the rupture of the connections of this system. Examples of that conclusion can be found in training systems, financial and economic systems, and technical systems. In this regard, we propose the following definition

Definition 2 / Risk systems of the system S in accordance with model G

Let the system S be modeled by a group of factors $\boldsymbol{G} = \langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I\rangle$ with generators $\{a_1, \dots, a_n | i \in I\}$, and defining relations $\{w_i(a_1, \dots, a_n) = 1, i \in I$. Then the system with intermittent connections simulated by the groups $\boldsymbol{G_J}$=$\langle G = \{a_1, \dots, a_n | i \in I\} | \{w_i(a_1, \dots, a_n) = 1, i \in I \setminus J\rangle, J \subseteqq I$ is modeling subsystem risk of the system's model of group $\boldsymbol{G}$ for every $J \subseteqq I$, by a model $\boldsymbol{G}$.

In accordance with this definition one can construct the definition of a quasi – fractal risk system's S model

Definition 3 Let the system S be modeled by a quasi – fractal group of factors

$QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{G}^{\alpha}_k = (\langle G^{\alpha}_k; \Omega_k = \langle \cdot, \ ^{-1}, e\rangle \rangle, \alpha \in \Lambda_k))$,

$G^{\alpha}_k = \langle G^{\alpha}_k = \{a_{1k}^{\ \alpha}, \dots, a_{nk}^{\ \alpha} | i \in I\} | \{w_{ik}^{\ \alpha}(a_{1k}^{\ \alpha}, \dots, a_{nk}^{\ \alpha}) = 1, i \in I^{\alpha}_k\rangle$ with generators $\{a_{1k}^{\ \alpha}, \dots, a_{nk}^{\ \alpha} | i \in I\}$ and defining relations $\{w_{ik}^{\ \alpha}(a_{1k}^{\ \alpha}, \dots, a_{nk}^{\ \alpha}) = 1, i \in I^{\alpha}_k;$ here ω_γ is an ordinal.

Then the quasi – fractal system with intermittent connections simulated by the quasi – fractal groups

$QF^{\alpha}_{k<\omega_\gamma}(\boldsymbol{G_J}_k^{\ \alpha} = (\langle G_{J_k}^{\ \alpha}; \Omega_k = \langle \cdot, \ ^{-1}, e\rangle \rangle, \alpha \in \Lambda_k))$,

$G_{J_k}^{\ \alpha} = \langle G_{J_k}^{\ \alpha} = \{a_{1k}^{\ \alpha}, \dots, a_{nk}^{\ \alpha} | i \in I\} | \{w_{ik}^{\ \alpha}(a_{1k}^{\ \alpha}, \dots, a_{nk}^{\ \alpha}) = 1, i \in I^{\alpha}_k \setminus J^{\alpha}_k\rangle$ with generators $\{a_{1k}^{\ \alpha}, \dots, a_{nk}^{\ \alpha} | i \in I\}$ and defining relations $\{w_{ik}^{\ \alpha}(a_{1k}^{\ \alpha}, \dots, a_{nk}^{\ \alpha}) = 1, i \in I^{\alpha}_k \setminus J^{\alpha}_k$ is modeling subsystem risk of the system's model of group $\boldsymbol{G}$ for every $J \subseteqq I$, here ω_γ is an ordinal. The advantages of the latter definition run as follows. Using quasi – fractal technic one can construct algorithms one can improve a risk analysis algorithm for smart planning systems and a risk analysis algorithm for smart control systems, (Serdyukova, Serdyukov, 2022; Serdyukova, Serdyukov, Kusminova, Kusnetsov, Shishkina, 2022).

2.3 Risk Models Descriptions Using Homological Algebra Methods

Using Homological algebra's methods one can connect the notion of the state of the complex system S modeled by the group of factors $\boldsymbol{G_S}$ and a risks system $\boldsymbol{G_{Sj}}$ of the system S in accordance with the model $\boldsymbol{G_S}$.

In (Serdyukov, Serdyukova, Shishkina, 2023) the fisical sense of a group $Hom(G_S, A)$ in accordance with the notion of the state of a system S is considered. In the present section we shall accept the following: $Aut(G_S)$ is a model of the system's S states by a model $\boldsymbol{G_S}$. Using

the universal property of abelianization of a group G which runs as follows:

$$\begin{tikzcd} G \arrow[r, "ab"] \arrow[dr, "f"'] & G/[G,G] \arrow[r] \arrow[d, dashed, "F"] & 1 \\ & A & \end{tikzcd}$$

here

$$f = F \cdot ab$$
$$(\forall f\colon G \to A)(\exists F\colon G/[G,G] \to A)(f = F \cdot ab)$$
$$f = F \cdot ab \qquad (1.3)$$

one gets that any other group with this property is isomorphic to $G/[G,G]$.

This property allows one to establish a one-to-one correspondence between all homomorphisms from the group G to some abelian group A and all homorphisms from abelianization $G/[G,G]$ into group A. It allows us to connect a model of the system's S states by a model $\boldsymbol{G_S}$ and a model of the risks $\boldsymbol{G_{Sj}}$ of the system's S states by a model $\boldsymbol{G_S}$.

In the present section we shall accept the following: $Aut(G_S)$ is a model of the system's S states, $Hom(G_S, A_S)$ is a mapping/≡description of the system's S states through the model A_S, $Hom(G_S, A_R)$ is a mapping /≡description of the system's S states through the model A_R of the system R. We shall connect the homology group with topological distortions of the states of the system S, determined by different models of groups of factors that determine the system. So, from (1) we have

Corollary 4

The description of the system's S states through a model A_S, which is an abelian group

$$Hom(G_S, A_S)$$

coincides with the description of the system's S states through an abelianization $G_S\,/[G_S\,, G_S\,]$ of the group G_S by the model A_S.

Corollary 5

The description of the system's S risks states by a model $\boldsymbol{G_{Sj}}$ through a model A_S, which is an abelian group

$$Hom(\boldsymbol{G_{Sj}}, A_S)$$

coincides with the description of the system's S states through an abelianization $\boldsymbol{G_{Sj}}/[\boldsymbol{G_{Sj}}\,, \boldsymbol{G_{Sj}}]$ of the group $\boldsymbol{G_{Sj}}$ by the model A_S.

2.4 Closed Associative System's $\boldsymbol{S}$ and Risk Sytem's $\boldsymbol{G_J}$ Synergistic Effects Synchronization in Accordance with Molel of Factors $\boldsymbol{G_S}$ of the system $\boldsymbol{S}$

Information about the synergistic effects of the system and information about the synergistic effects of the system and its internal risks are of great importance for the smooth functioning of complex systems.

In this section the we shall use the prime number theorem (PNT) to describe the number of synergistic effects of a closed associative system modelled by a finite group. Previously, in [4, Ch 3, Ch 4, Ch 10], we have shown that if system S is modeled by a finite group $\boldsymbol{G_S}$ of factors defining S, then the number of synergetic effects of S is a divisor of $|\boldsymbol{G_S}\,|$. So, to describe finite system S without synergetic effects, one need the prime number theorem (PNT), as the number of elements of a finite system S without synergetics effects should be prime. So, the model of risks $\boldsymbol{G_{Sj}}$ of a system S in accordance with a group of factors $\boldsymbol{G_S}$ is either unit group or coincides with $\boldsymbol{G_S}$.

The prime number theorem (PNT) describes the asymptotic distribution of the prime numbers among the positive integers. It formalizes the idea that primes become less common as they become larger by precisely quantifying the rate at which this occurs.

It is well known, that the distribution function of primes, is a function equal to the number of primes less than or equal to the real number x.

The distribution of primes is described more precisely by the integral logarithm function. The logarithmic integral function or integral logarithm $li(x) = \int_0^x \frac{dt}{lnt}, x \neq 1,$ is a special function which has number theoretic significance. In particular, according to the prime number theorem, it is a very good approximation to the prime-counting function, which is defined as the number of prime numbers less than or equal to a given value X.

The integral logarithm and the integral exponential function are related by the relation:

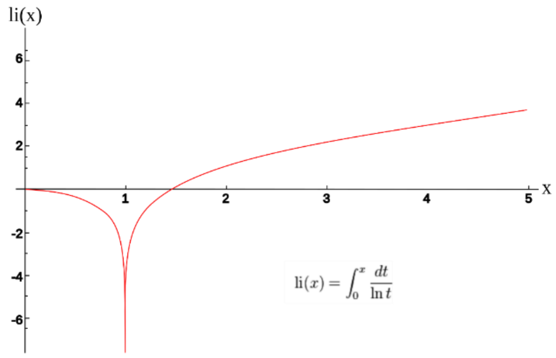

Figure 1: The logarithmic integral function (integral logarithm).

Fig. 1 shows that an increase in the number of factors of a group $\boldsymbol{G_S}$, which models the system S, the number of subsystems of the system S without synergetics effects increases.

So, we have

Theorem 6 (on the distribution of the number of synergetic effects in the systems)

The number of factors $|\boldsymbol{G_S}|$ of a group $\boldsymbol{G_S}$, which models the system S without synergetics effects, yield the prime number theorem (PNT). So, for the model of risks $\boldsymbol{G_{Sj}}$ of a system S in accordance with a group of factors $\boldsymbol{G_S}$ the number of risks' factors $|\boldsymbol{G_{Sj}}|$ increases with the growth of $|\boldsymbol{G_S}|$, but $\boldsymbol{G_{Sj}}$ has no synergetics effects.

Theorem 7.

The number of subsystems without synergetics effects of a closed system S, modeled by a group $\boldsymbol{G_S}$ of factors defining the system S, increases with the growth the number of factors $|\boldsymbol{G_S}|$, which determine the system S. So, for the model of risks $\boldsymbol{G_{Sj}}$ of a system S in accordance with a group of factors $\boldsymbol{G_S}$ the number of risks' subsystems $\boldsymbol{G_{Sj}}$ increases with the grouth of $|\boldsymbol{G_S}|$, but the model of risks $\boldsymbol{G_{Sj}}$ by the model $\boldsymbol{G_S}$ has no synergetics effects.

3 CONCLUSIONS

Development of a methodology for constructing algebraic quasi-fractal models of smart systems, and, on this basis, the creation of practical applications in the field of digital transformation of the economy and finance, including in the field of taxation is one of the main tasks

The main scientific results in this area run as follows

The concept of algebraic formalization of a smart system was introduced (Serdyukova, 2018).

1. The concept of a presystem was introduced in (Serdyukova, Serdyukov, 2021);
2. The means of quasi-fractal mathematical logic are defined and described, allowing one to compare deterministic and random chaos (Serdyukova, Serdyukov, 2021);
3. Approaches to obtaining results using methods of quasi-fractal mathematical logic have been developed (Serdyukova, 2018; Serdyukova, Serdyukov, 2021; Serdyukova, 2018; Serdyukova, Serdyukov, 2022; Serdyukova, Serdyukov, 2019; Serdyukova, Serdyukov, Kusminova, Shishkina, 2024; Serdyukova, Serdyukov, Kusminova, Kusnetsov, Shishkina, 2022; Serdyukova, 1991; Serdyukov, Serdyukova, Shishkina, 2023);
4. Functions for digitalization of propositional algebra and narrow predicate calculus (UPC) have been developed (Serdyukova, Serdyukov, 2019);
5. Methods for algebraic formalization of smart systems and their algebraic identification have been developed, and their practical applications have been found (Serdyukova, 2018; Serdyukova, 2018; Serdyukova, Serdyukov, 2022; Serdyukova, Serdyukov, 2019; Serdyukova, Serdyukov, Kusminova, Shishkina, 2024; Serdyukova, Serdyukov, Kusminova, Kusnetsov, Shishkina, 2022; Serdyukova, 1991; Serdyukov, Serdyukova, Shishkina, 2023).

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