Z-Regression for Analysis and Reliable Prediction of Qualitative Characteristics

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Abstract: A regression model under Z-information has developed in the paper. The initial information for constructing the model is linguistic Z-numbers, defined on the unit segment. For a Z-number, an interval type-2 fuzzy number is defined, using aggregating characteristics of reliability (second component of Z-number). To do this, the aggregating segment of reliability and its middle are use. In the paper was made a choice in favor of transforming Z-numbers into type-2 fuzzy numbers, since Z-numbers are closer in structure to them than to type-1 fuzzy numbers, which, however, are most often used to transform Z-numbers and construct different models. Input and output interval type-2 fuzzy numbers are determined for the initial Z-numbers. The optimization function is defined as the sum of squares for the differences between parameters of output type-2 fuzzy numbers and the model type-2 fuzzy numbers. To determine the unknown regression coefficients, the minimum of the optimization function was found using known methods. Reliability recognition of model output information has developed.

1 INTRODUCTION

Quantitative and qualitative characteristics are used to assess the green infrastructure of cities. For example, the height of a tree or the leaf area of a tree are quantitative characteristics, while the state of plants or satisfaction with a green area are qualitative characteristics. It is quite difficult to analyze qualitative characteristics and predict their values, since arithmetic operations for their values are not defined.

If the values of qualitative characteristics are estimated with a certain level of reliability, then the tasks of their analysis and prediction become even more complicated. For example, when assessing the state of green spaces, even an experienced expert cannot be absolutely sure of the reliability of his data, and therefore provides not only his assessment - "No signs of weakness", but also the reliability of this assessment - "Very likely". To analyze and predict data with fuzzy estimates of the values of qualitative characteristics, as well as with fuzzy reliability of these estimates, a special mathematical apparatus based on regression analysis and Z-numbers is required.

Regression analysis plays a key role in the analysis of relationships between input and output qualitative information, as well as in prediction problems. This analysis is especially important when processing Z-information (information with Z-numbers or information in which fuzzy data is presented with a certain level of reliability) (Zadeh, 2011).

To be able to analyze Z-information, arithmetic operations have been developed (Zadeh, 2012; Kang, Wei, Li, Deng, 2012;Kang, Wei, Li, Deng, 2012; Aliev, Alizadeh, Huseynov, 2015; Aliev, Huseynov, Zeinalova, 2016; Aliev, Huseynov, Aliyeva, 2015) and Z-numbers have been used to formalize expert reasoning (Yager, 2012). Distances between Z-numbers and ways to rank them have been determined (Wang, Mao, 2019; Poleshchuk, 2019; Aliyev, Talal Mraizid, Huseynov, 2015; Aliyev, 2015). Methods for solving applied decision-making problems under Z-information have been developed (Kang, Wei, Li, Deng, 2012; Wang, Mao, 2019; Poleshchuk, 2019; Aliyev, Talal Mraizid, Huseynov, 2015; Poleshchuk, 2023).

Regression models were urgently needed to predict and study dependencies in the context of Z-information. The first models appeared in works

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(Sadikoglu, Huseynov, Memmedova, 2016; Zeinalova, Huseynov, Sharghi, 2017), which were built on the basis of operations with Z-numbers and probability distributions of random variables. The optimization problem for finding the unknown coefficients (ordinary numbers) of the regression model was posed using the Jaccard measure.

The problem was that in defining the Z-number, Professor Lotfi Zadeh did not limit the Z-number to considering only the probability distributions of random variables in determining the reliability of its first component. Therefore, this limitation was supposed to be removed so that it would be possible to use a regression model for any Z-information without limitation.

This limitation was removed in (Poleshchuk, 2021), where a linear regression model with fuzzy coefficients was developed. The author has moved away from direct operation with Z-numbers (which is always quite difficult) and has defined aggregating segments for the input and output information represented by Z-numbers. With the help of these aggregating segments, operations on Z-numbers and distances between them were determined, an optimization problem was posed and solved, and output Z-numbers were recognized.

The next regression model under Z-information was nonlinear regression with crisp coefficients, developed in (Poleshchuk, 2022). For its development, the basics of the paper (Poleshchuk, 2021) were used.

Both models of papers (Poleshchuk, 2021; Poleshchuk, 2022) were based on a single approach to the step-by-step transformation of both components of Z-numbers into aggregation segments with subsequent operation on them, and on a single approach to the formulation of optimization problems. Therefore, the question arose of developing a model based on a different approach to the presentation of initial data and the formulation of an optimization problem for the purpose of comparative analysis and the selection of the best model for solving practical problems.

In (Poleshchuk, 2023), a linear regression model under Z-information with coefficients in the form of fuzzy triangular numbers was constructed. In the paper, it was defined the loss function for the difference between the initial output Z-number and the model output Z-number. The optimization problem was solved from the minimum condition of the sum of loss functions for the differences between the initial output Z-numbers and the model output Z-numbers. The solution of the optimization problem was reduced to the solution of the well-known linear programming problem.

If you look deeper, then one way or another, when constructing regression models, there is a transformation from Z-numbers to type-1 fuzzy numbers (ordinary fuzzy numbers). However, Z-numbers in their essence and structure are closer to type-2 fuzzy numbers, as evidenced by studies, for example, in the paper (Aliev, Guirimov, 2018).

Based on this, when constructing a regression model and using type-2 fuzzy numbers that are closer to Z-numbers compared to type-1 fuzzy numbers, we should expect to retain more information about the initial information and improve the quality of the model. Therefore, this paper aims to develop a regression model under Z-information using the relationships between Z-numbers and type-2 fuzzy numbers, described and proven in the paper (Aliev, Kreinovich, 2017).

Section 2 of the paper gives the basic concepts and definitions. Section 3 proposes a regression model under Z-information. Section 4 gives conclusions.

2 BASIC CONCEPTS AND DEFINITIONS

Consider a trapezoidal type-1 fuzzy number $\tilde{A} = (a_1, a_2, a_L, a_R)$:

$$\tilde{A} = \begin{cases} \frac{x-a_1+a_L}{a_L}, a_1 - a_L \le x \le a_1, a_L \ge 0, \\ 1, a_1 \le x \le a_2 \\ \frac{a_2+a_R-x}{a_R}, a_2 \le x \le a_2 + a_R, a_R \ge 0. \end{cases}$$

Then α -cut of $\tilde{A}$ is A_α such that:

$$A_\alpha = \{x \in R: \mu_A(x) \ge \alpha\} = [a - (1-\alpha)a_L, a + (1-\alpha)a_R], \alpha \in [0,1].$$

In (Domrachev, Poleshuk, 2003), based on A_α an aggregating segment $[\beta_1, \beta_2]$ for fuzzy number $\tilde{A} = (a_1, a_2, a_L, a_R)$ is determined:

$$\beta_1 = \int_0^1 \frac{2a_1 - (1-\alpha)a_L}{2} 2\alpha \, d\alpha = a_1 - \frac{1}{6}a_L, \quad (1)$$

$$\beta_2 = \int_0^1 \frac{2a_2 - (1-\alpha)a_R}{2} 2\alpha\, d\alpha = a_1 - \frac{1}{6}a_R.$$

In (Poleshchuk, 2023), a weighted point β for a fuzzy number $\tilde{A} = (a_1, a_2, a_L, a_R)$ according to (1) is determined as follows:

$$\beta = \frac{\beta_1+\beta_2}{2} = \frac{a_1+a_2}{2} + \frac{a_R-a_L}{12}. \quad (2)$$

A linguistic variable X with terms $T(X) = \{X_l,\ l = \overline{1,m}\}$ is called a collection $\{X, T(X), U, V, S\}$. Term names $X_l, l = \overline{1,m}$ are determined by the rule V and the corresponding fuzzy sets of U for fuzzy variables $\tilde{X}_l, l = \overline{1,m}$ are determined by the rule S (Zadeh, 1975).

The construction of linguistic variables with the properties described in (Ryjov, 1994) is described in detail in (Poleshchuk, 2018).

An interval type-2 fuzzy number (IT2 FS) $\tilde{A}$ is defined by lower fuzzy number (LFN) $\underline{\tilde{A}} = (a_1^L, a_2^L, a_L^L, a_R^L)$ and upper fuzzy number (UFN) $\overline{\tilde{A}} = (a_1^U, a_2^U, a_L^U, a_R^U)$ (Liu, Mendel, 2009).

In the paper (Poleshchuk, Komarov, 2012) for LFN $\underline{\tilde{A}} = (a_1^L, a_2^L, a_L^L, a_R^L)$ and UFN $\overline{\tilde{A}} = (a_1^U, a_2^U, a_L^U, a_R^U)$ IT2 FS $\tilde{A}$ are defined respectively aggregating segments $[\beta_1^L, \beta_2^L]$, $[\beta_1^U, \beta_2^U]$ in accordance with (1). For two IT2 FSs $\tilde{A}, \tilde{B}$ with aggregating segments $[\beta_1^L, \beta_2^L]$, $[\beta_1^U, \beta_2^U]$, $[\delta_1^L, \delta_2^L]$, $[\delta_1^U, \delta_2^U]$ the distance $d(\tilde{A}, \tilde{B})$ is defined in the same paper:

$$d(\tilde{A}, \tilde{B}) = \sqrt{(\beta_1^L - \delta_1^L)^2 + (\beta_2^L - \delta_2^L)^2 + (\beta_1^U - \delta_1^U)^2 + (\beta_2^U - \delta_2^U)^2}. \quad (3)$$

Z-number is defined in (Zadeh, 2011) as a pair $Z = (\tilde{C}, \tilde{R})$, where $\tilde{C}$, $\tilde{R}$ are fuzzy numbers and $\tilde{R}$ is reliability of $\tilde{C}$.

A Z-number whose components are the values of linguistic variables is called a linguistic Z-number (Ding, Zhu, Lu, Wang, Feng, 2020).

In (Poleshchuk, 2023), a weighted fuzzy number $\tilde{\mathcal{A}}$ for the Z-number $Z = (\tilde{A}, \tilde{R})$, $\tilde{A} = (a_1, a_2, a_L, a_R)$, $\tilde{R} = (r_1, r_2, r_L, r_R)$ is determined as follows: $\tilde{\mathcal{A}} = (\gamma a_1, \gamma a_2, \gamma a_L, \gamma a_R)$, where $\gamma = \frac{r_1+r_2}{2} + \frac{r_R-r_L}{12}, \gamma > 0$ is a weighted point for the fuzzy number $\tilde{R} = (r_1, r_2, r_L, r_R)$.

3 PROBLEM FORMULATION AND SOLUTION

Let's denote the initial input data with $Z_{ij} = (\tilde{X}_{ij}, \tilde{R}_{ij}), i = \overline{1,m}, j = \overline{1,n}$ and the initial output data with $Z_j = (\tilde{Y}_j, \tilde{R}_j), j = \overline{1,n}$, $\tilde{X}_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^L, x_{ij}^R), i = \overline{1,m}, j = \overline{1,n}$, $\tilde{Y}_j = (y_j^1, y_j^2, y_j^L, y_j^R), j = \overline{1,n}$, $\tilde{R}_{ij} = (r_{ij}^1, r_{ij}^2, r_{ij}^L, r_{ij}^R)$, $\tilde{R}_j = (r_j^1, r_j^2, r_j^L, r_j^R), i = \overline{1,m}, j = \overline{1,n}$.

The initial Z-numbers are linguistic Z-numbers with universal sets U=[0,1]. This means that the first components of Z-numbers are formalizations of linguistic scales used to assess the input qualitative characteristics $X_i, i = \overline{1,m}$ и and the output qualitative characteristic Y.

Each of fuzzy number $\tilde{R}_{ij}, \tilde{R}_j, i = \overline{1,m}, j = \overline{1,n}$ equals to one of value $\tilde{R}_v, v = \overline{1,V}$ a linguistic variable «Reliability».

Let construct a regression model in the form:

$$Z = a_1 Z_1 + a_2 Z_2 + \cdots a_m Z_m, \quad (4)$$

Where $a_i, i = \overline{1,m}$ - crisp numbers.

Using the relationship between Z-numbers and type-2 fuzzy numbers (T2 FNs), described and proven in the paper (Aliev, Kreinovich, 2017), we transform Z-number $Z = (\tilde{A}, \tilde{R})$, $\tilde{A} = (a_1, a_2, a_L, a_R)$, $\tilde{R} = (r_1, r_2, r_L, r_R)$ into IT2 FN $\tilde{\mathbb{A}}$, defined by LFN $\underline{\tilde{\mathbb{A}}} = \left(a_1, \frac{\beta_1+\beta_2}{2}a_2, a_L, \frac{\beta_1+\beta_2}{2}a_R\right)$ and UFN $\overline{\tilde{\mathbb{A}}} = (\beta_1 a_1, a_2, \beta_1 a_L, a_R)$ where $[\beta_1, \beta_2]$ an aggregating segment for fuzzy number $\tilde{R} = (r_1, r_2, r_L, r_R)$ (reliability for $\tilde{A} = (a_1, a_2, a_L, a_R)$).

In the paper (Poleshchuk, 2023), Z-number $Z = (\tilde{A}, \tilde{R})$, $\tilde{A} = (a_1, a_2, a_L, a_R)$, $\tilde{R} = (r_1, r_2, r_L, r_R)$ transformed into ordinary fuzzy number $\tilde{\mathcal{A}} = \left(\frac{\beta_1+\beta_2}{2}a_1, \frac{\beta_1+\beta_2}{2}a_2, \frac{\beta_1+\beta_2}{2}a_L, \frac{\beta_1+\beta_2}{2}a_R\right)$.

As $\beta_1 < \frac{\beta_1+\beta_2}{2} < 1, \frac{\beta_1+\beta_2}{2} < 1, 0 \le \beta_1 \le 1, 0 \le \beta_2 \le 1, \beta_2 > \beta_1$, then the values of the membership function of the number $\tilde{\mathcal{A}} = \left(\frac{\beta_1+\beta_2}{2}a_1, \frac{\beta_1+\beta_2}{2}a_2, \frac{\beta_1+\beta_2}{2}a_L, \frac{\beta_1+\beta_2}{2}a_R\right)$ belong to the values of the membership function of the number $\tilde{\mathbb{A}}$, defined by LFN $\underline{\tilde{\mathbb{A}}} = \left(a_1, \frac{\beta_1+\beta_2}{2}a_2, a_L, \frac{\beta_1+\beta_2}{2}a_R\right)$ and UFN $\overline{\tilde{\mathbb{A}}} = (\beta_1 a_1, a_2, \beta_1 a_L, a_R)$.

Thus, the representation of the Z-number $Z = (\tilde{A}, \tilde{R}),\quad \tilde{A} = (a_1, a_2, a_L, a_R),\ \tilde{R} = (r_1, r_2, r_L, r_R)$ in the paper (Poleshchuk, 2023) as an ordinary fuzzy number $\tilde{\mathcal{A}} = \left(\frac{\beta_1+\beta_2}{2}a_1, \frac{\beta_1+\beta_2}{2}a_2, \frac{\beta_1+\beta_2}{2}a_L, \frac{\beta_1+\beta_2}{2}a_R\right)$ can be considered as a special case of representing the Z-number в виде IT2 FN $\tilde{\mathbb{A}}$, defined by LFN $\underline{\tilde{\mathbb{A}}} = \left(a_1, \frac{\beta_1+\beta_2}{2}a_2, a_L, \frac{\beta_1+\beta_2}{2}a_R\right)$ and UFN $\overline{\tilde{\mathbb{A}}} = (\beta_1 a_1, a_2, \beta_1 a_L, a_R)$.

Define IT2 FNs $\tilde{\mathbb{X}}_{ij}, i = \overline{1,m}, j = \overline{1,n}$ with LFN $\underline{\tilde{\mathbb{X}}_{ij}}$ and UFN $\overline{\tilde{\mathbb{X}}_{ij}}$ for input Z-numbers $Z_{ij} = (\tilde{X}_{ij}, \tilde{R}_{ij}), i = \overline{1,m}, j = \overline{1,n},\ \tilde{X}_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^L, x_{ij}^R), = \overline{1,m}, j = \overline{1,n}, \tilde{R}_{ij} = (r_{ij}^1, r_{ij}^2, r_{ij}^L, r_{ij}^R), i = \overline{1,m}, j = \overline{1,n}$:

$$\underline{\tilde{\mathbb{X}}_{ij}} = \left(x_{ij}^1, \frac{\beta_{ij}^1+\beta_{ij}^2}{2}x_{ij}^2, x_{ij}^L, \frac{\beta_{ij}^1+\beta_{ij}^2}{2}x_{ij}^R\right), \overline{\tilde{\mathbb{X}}_{ij}} = \left(\beta_{ij}^1 x_{ij}^1, x_{ij}^2, \beta_{ij}^1 x_{ij}^L, x_{ij}^R\right), \quad (5)$$

where $[\beta_{ij}^1, \beta_{ij}^2]$ are aggregating segments for fuzzy numbers $\tilde{R}_{ij} = (r_{ij}^1, r_{ij}^2, r_{ij}^L, r_{ij}^R), i = \overline{1,m}, j = \overline{1,n}$.

Define IT2 FNs $\tilde{\mathbb{Y}}_j, j = \overline{1,n}$ with LFN $\underline{\tilde{\mathbb{Y}}_j}$ and UFN $\overline{\tilde{\mathbb{Y}}_j}$ for output Z-numbers $Z_j = (\tilde{Y}_j, \tilde{R}_j), j = \overline{1,n}, \tilde{Y}_j = (y_j^1, y_j^2, y_j^L, y_j^R), \tilde{R}_j = (r_j^1, r_j^2, r_j^L, r_j^R)$:

$$\underline{\tilde{\mathbb{Y}}_j} = \left(y_j^1, \frac{\beta_j^1+\beta_j^2}{2}y_j^2, y_j^L, \frac{\beta_j^1+\beta_j^2}{2}y_j^R\right), \overline{\tilde{\mathbb{Y}}_j} = \left(\beta_j^1 y_j^1, y_j^2, \beta_j^1 y_j^L, y_j^R\right), \quad (6)$$

where $[\beta_j^1, \beta_j^2]$ are aggregating segments for fuzzy numbers $\tilde{R}_j = (r_j^1, r_j^2, r_j^L, r_j^R), j = \overline{1,n}$.

Define IT2 FNs $\tilde{\mathbb{Y}}_{M_j}, j = \overline{1,n}$ f with LFN $\underline{\tilde{\mathbb{Y}}_{M_j}}$ and UFN $\overline{\tilde{\mathbb{Y}}_{M_j}}$ $f$or the model output Z-numbers $\overline{Z_j} = \sum_{i=1}^m a_i Z_{ij}, j = \overline{1,n},$ using (5):

$\underline{\tilde{\mathbb{Y}}_{M_j}} = \left(\sum_{i=1}^m a_i\, x_{ij}^1, \sum_{i=1}^m a_i \frac{\beta_{ij}^1+\beta_{ij}^2}{2}x_{ij}^2, \sum_{i=1}^m a_i\, x_{ij}^L, \sum_{i=1}^m a_i \frac{\beta_{ij}^1+\beta_{ij}^2}{2}x_{ij}^R\right),$
$a_i > 0, i = \overline{1,m}, j = \overline{1,n}$

$\overline{\tilde{\mathbb{Y}}_{M_j}} = \left(\sum_{i=1}^m a_i\, \beta_{ij}^1 x_{ij}^1, \sum_{i=1}^m a_i x_{ij}^2, \sum_{i=1}^m a_i\, \beta_{ij}^1 x_{ij}^L, \sum_{i=1}^m a_i x_{ij}^R\right)$
$, a_i < 0, i = \overline{1,m}, j = \overline{1,n}.$

In general,

$\underline{\tilde{\mathbb{Y}}_{M_j}} = \left(\sum_{i=1}^m a_i\, x_{ij}^{pi}, \sum_{i=1}^m a_i \frac{\beta_{ij}^1+\beta_{ij}^2}{2}x_{ij}^{qi}, \sum_{i=1}^m a_i\, x_{ij}^{M_{pi}}, \sum_{i=1}^m a_i \frac{\beta_{ij}^1+\beta_{ij}^2}{2}x_{ij}^{M_{qi}}\right)$
$, j = \overline{1,n},$

$\overline{\tilde{\mathbb{Y}}_{M_j}} = \left(\sum_{i=1}^m a_i\, \beta_{ij}^1 x_{ij}^{pi}, \sum_{i=1}^m a_i x_{ij}^{qi}, \sum_{i=1}^m a_i\, \beta_{ij}^1 x_{ij}^{M_{pi}}, \sum_{i=1}^m a_i x_{ij}^{M_{qi}}\right), j = \overline{1,n},$

where

$$p_i = \begin{cases} 1, a_i > 0 \\ 2, a_i < 0, \end{cases} q_i = \begin{cases} 1, a < 0 \\ 2, a_i > 0, \end{cases} M_{p_i} = \begin{cases} L, p_i = 1 \\ R, p_i = 2, \end{cases} M_{q_i} = \begin{cases} L, q_i = 1 \\ R, q_i = 2, \end{cases} i = \overline{1,m}.$$

The optimization problem for finding the unknown coefficients $a_i < 0, i = \overline{1,m}$ is following:

$$\begin{aligned} F(a_1, a_2, \dots, a_m) = \sum_{j=1}^{n} \Bigg[& \left(y_j^1 - \sum_{i=1}^m a_i\, x_{ij}^{pi}\right)^2 \\ & + \left(\frac{\beta_j^1+\beta_j^2}{2}y_j^2 - \sum_{i=1}^m a_i \frac{\beta_{ij}^1+\beta_{ij}^2}{2}x_{ij}^{qi}\right)^2 \\ & + \left(y_j^L - \sum_{i=1}^m a_i\, x_{ij}^{M_{pi}}\right)^2 \\ & + \left(\frac{\beta_j^1+\beta_j^2}{2}y_j^2 - \sum_{i=1}^m a_i \frac{\beta_{ij}^1+\beta_{ij}^2}{2}x_{ij}^{qi}\right)^2 \\ & + \left(\beta_j^1 y_j^1 - \sum_{i=1}^m a_i\, \beta_{ij}^1 x_{ij}^{pi}\right)^2 \\ & + \left(y_j^2 - \sum_{i=1}^m a_i x_{ij}^{qi}\right)^2 \\ & + \left(\beta_j^1 y_j^L - \sum_{i=1}^m a_i\, \beta_{ij}^1 x_{ij}^{M_{pi}}\right)^2 \\ & + \left(y_j^R - \sum_{i=1}^m a_i x_{ij}^{M_{qi}}\right)^2 \Bigg] \to min. \end{aligned}$$

Function $F(a_1, a_2, \dots, a_m)$is piecewise differentiable function since $\sum_{i=1}^m a_i\, x_{ij}^{pi}$,

$$\sum_{i=1}^{m} a_i \frac{\beta_{ij}^1 + \beta_{ij}^2}{2} x_{ij}^{qi}, \sum_{i=1}^{m} a_i x_{ij}^{M_{pi}}, \sum_{i=1}^{m} a_i \frac{\beta_{ij}^1 + \beta_{ij}^2}{2} x_{ij}^{qi}, \sum_{i=1}^{m} a_i \beta_{ij}^1 x_{ij}^{pi}, \sum_{i=1}^{m} a_i x_{ij}^{qi},$$

$\sum_{i=1}^{m} a_i \beta_{ij}^1 x_{ij}^{M_{pi}}, \sum_{i=1}^{m} a_i x_{ij}^{M_{qi}}$ are piecewise linear functions. Unknown coefficients $a_1, a_2, \dots, a_m$ are found according to the method (Coleman, Li, 1996).

The first component of the model output Z -number is found using a regression model into which the first component of the input Z -number is substituted. To determine the reliability of the resulting fuzzy number (to determine the second component of the model output Z -number), it is necessary to compare IT2 FN $\underline{\mathbb{Y}_M} = (u_1^L, u_2^L, u_L^L, u_R^L)$, $\overline{\mathbb{Y}_M} = (u_1^U, u_2^U, u_L^U, u_R^U)$, into which the model Z -number is transformed, and IT2 FNs $\underline{\mathbb{Y}_v} = (u_{1v}^L, u_{2v}^L, u_{Lv}^L, u_{Rv}^L)$, $\overline{\mathbb{Y}_v} = (u_{1v}^U, u_{2v}^U, u_{Lv}^U, u_{Rv}^U)$, $v = \overline{1,V}$, into which Z -numbers are transformed, the first component of which is the first component of the model Z -number, and the second components are the formalization $\tilde{R}_v, v = \overline{1,V}$ of the values of a linguistic variable with name –«Reliability». If

$$\begin{aligned}\left(u_1^L - u_{1p}^L\right)^2 + \left(u_2^L - u_{2p}^L\right)^2 + \left(u_L^L - u_{Lp}^L\right)^2 \\ + \left(u_R^L - u_{Rp}^L\right)^2 + \left(u_1^U - u_{1p}^U\right)^2 \\ + \left(u_2^U - u_{2p}^U\right)^2 + \left(u_L^U - u_{Lp}^U\right)^2 \\ + \left(u_R^U - u_{Rp}^U\right)^2 \\ = \min_{v=\overline{1,V}} (u_1^L - u_{1v}^L)^2 + (u_2^L - u_{2v}^L)^2 \\ + (u_L^L - u_{Lv}^L)^2 + (u_R^L - u_{Rv}^L)^2 \\ + (u_1^U - u_{1v}^U)^2 + (u_2^U - u_{2v}^U)^2 \\ + (u_L^U - u_{Lv}^U)^2 + (u_R^U - u_{Rv}^U)^2,\end{aligned}$$

then the reliability of the second component of the model output Z -number is fuzzy number $\tilde{R}_p$ and accordingly p-th term of linguistic variable «Reliability».

4 CONCLUSIONS

To conduct environmental monitoring of cities, assess the qualitative characteristics of the state of green spaces and ecosystem services of urban green areas, experts are involved, for whom it is natural to use words and phrases of professional language that introduce fuzziness into the information coming from them. Since the most experienced expert cannot be absolutely sure of his estimates, it is necessary to take into account the reliability of these estimates when analyzing and predicting them. As a result, we receive information with fuzzy estimates and fuzzy reliability of these estimates, which must be analyzed and predicted in order to develop timely control actions.

For this purpose, the paper has developed a regression model, the input and output information of which are Z-numbers. The proven property of proximity of the structures Z -numbers and type-2 fuzzy numbers was used in the construction. Input and output interval type-2 fuzzy numbers are determined for the initial Z-numbers. The optimization function is defined as the sum of squares for the differences between parameters of output type-2 fuzzy numbers and the model type-2 fuzzy numbers.

To predict the output information, a model has been developed for recognizing its reliability and presenting it as the value of the scale used to evaluate it.

The Z-regression developed in the paper opens up new opportunities for the analysis and predicting of qualitative information, taking into account its reliability in monitoring the environmental state of cities for the development of timely control actions.

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