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Analysis and Mathematical Modeling of the Dynamics of the Spread of Morbidity from Medical and Preventive Measures

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Abstract: The article is devoted to the analysis and mathematical modeling of the dynamics of the spread of diseases from medical and preventive measures. During mass, preventive examinations (screening), provision of modern high-tech medical equipment certainly increases the effectiveness of diagnostics. This paper considers a discrete model of disease spread growth based on the logistic equation and the Sletkov-Arzamastsev spatial model. Key aspects of the study are numerical modeling, sensitivity analysis to parameters such as growth and diffusion coefficients, and visualization of the spatial distribution of the disease. Particular attention is paid to the interaction between the spread of the disease and treatment and preventive measures (TPM), modeled through probabilistic collisions, energy accumulation, and diffusion processes. The work has practical significance in the field of mathematical modeling of disease spread and healthcare organization.$\overline{r}\overline{D}$

1 INTRODUCTION

Caring for the health of citizens is a strategic task of the state, the solution of which lies in the prevention and diagnosis of diseases and the development of appropriate means

It is known that it is easier to prevent a disease than to treat it later. According to the recommendations of the World Health Organization, prevention includes the following aspects:

- primary, which is aimed at preventing the expression of risk factors in people with excellent health;
- secondary, which involves combating existing risk factors;
- tertiary, which carries out active therapy of developing and developed diseases to prevent the development of the inflammatory process and the occurrence of complications.

As a result, the national project "Long and Active Life" places special emphasis on "strengthening the preventive focus", and early detection of inflammatory processes is a priority task of this project. The main influence in the project "Long and Active Life" is given to strengthening the primary health care diagnostic services, as well as equipping medical institutions with the latest modern equipment and providing the country's population with high-tech qualified medical care. It is assumed that there will be a mass preventive examination (or screening) of the population, in which each resident of our country must receive a so-called "health passport" and undergo an electrocardiographic and fluorographic examination annually.

The main risk factors for the development of the disease are divided into two groups:

- factors that cannot be changed, the so-called "individual" factors. These include gender, age and heredity;
- factors that can be corrected, the so-called "social" factors. These include
- access to health care. The level of access to health care and preventive measures can vary greatly depending on social status and economic situation. People with low incomes or without health insurance may have limited access to vaccinations, treatment

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and medical care. This contributes to the spread of diseases;

- population density. In cities with high population density, where people live closer to each other, diseases can spread faster. This is especially true for airborne infections;

- social contacts and behavior. Attending mass events, traveling, improper use of antiseptics and unwillingness to observe hygiene measures. This can contribute to the transmission of diseases from person to person;

- education and information. Education and information about diseases play an important role in preventing and controlling their spread. People who know about preventive measures and follow them have a better chance of preventing infection;

- economic status. Economic inequalities can affect people's ability to receive health care and maintain hygiene standards. Economically vulnerable groups may be more likely to become ill;

- Migration and mobility. Travel and migration can contribute to the global spread of diseases. People crossing borders can carry diseases from one country to another;

- public health. The level of sanitation, cleanliness and quality of water supply in public places is important for controlling the spread of infections;

- Vaccination and public opinion. Opinions about vaccination can vary greatly within society. Insufficient vaccination can create conditions for outbreaks of disease.

The aim of the article is to analyze and mathematically model the dynamics of the spread of diseases from medical and preventive measures.

To achieve the goal, the following tasks are solved:

- improvement of the mathematical model of the growth of morbidity on the plane;

- analysis of parameters of treatment and preventive measures;

- mathematical modeling of the growth of morbidity on a plane;

- analysis of the sensitivity of the solution to changes in the parameters of treatment and preventive measures;

- visualization of solutions.

2 MATERIALS AND METHODS

2.1 Sletkov-Arzamastsev model

The study of the dynamics of the spread of the disease is of key importance for the organization and implementation of treatment and preventive measures. As a discrete model, the authors chose the Sletkov-Arzamastsev model, which takes into account the spatial distribution and interaction of microorganism colonies. This model provides a tool for studying complex biological systems: The Sletkov-Arzamastsev model is described by the equation:

$$N_{i,j}^{t+1} = N_{i,j}^{t} + r * N_{i,j}^{t} * \left(1 - \frac{N_{i,j}^{t}}{K}\right) - \mu * N_{i,j}^{t} + D * \sum_{\text{соседи}} \left(N_{\text{сосед}}^{t} - N_{i,j}^{t}\right),$$

Where:

$N_{i,j}^{t}$ – the number of cases in cell (i, j) at step t,

K=200 – capacity of the environment;

D=0.1 – diffusion coefficient;

μ=0.05 — mortality.

In the course of solving the Sletkov-Arzamastsev model, the logistic equation is used (Murray, 5.11):

$$x_{n+1} = r * x_n * (1 - x_n),$$

Where:

xn∈[0,1] – normalized population size;

r∈(0.4] – growth coefficient.

The logistic equation describes limited population growth, where the rate of reproduction decreases as the carrying capacity is approached.

Discrete form of the logistic equation:

$x_{n+1} = r * x_n * (1 - x_n)$

Stability analysis of the logistic equation:

Fixed points:

Solutions of the equation x=r·x·(1−x):

x* = 0 (trivial point),

x* =$1 - \frac{1}{r}$(non-trivial point).

Stability:

For 1 < r < 3: The non-trivial point is stable.

When r > 3: Bifurcations and chaos occur.

Thus, the improvement of the Sletkov-Arzamastsev model consists in combining:

1. Logistic growth in each grid cell:

$$\Delta N_{\text{рост}} = r * N_{i,j}^{t} * \left(1 - \frac{N_{i,j}^{t}}{K}\right).$$

2. Diffusions between adjacent cells (Laplacian approximation):

$$\Delta N_{\text{диффузия}} = D * (N_{i-1,j}^{t} + N_{i+1,j}^{t} + N_{i,j-1}^{t} + N_{i,j+1}^{t} - 4N_{i,j}^{t})$$

3. Mortality accounting:

$$\Delta N_{\text{смертность}} = -\mu * N_{i,j}^{t}$$

Final equation:

$$N_{i,j}^{t+1} = N_{i,j}^{t} + \Delta N_{\text{рост}} + \Delta N_{\text{диффузия}} + \Delta N_{\text{смертность}}$$

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3 RESULTS AND DISCUSSION

3.1 Processing methods

Normalization:

$$N_{norm} = \frac{N - N_{min}}{N_{max} - N_{min}}$$

Noise filtering:

Moving average with a window of 3 iterations:

$$\widetilde{N_t} = \frac{N_{t-1} + N_t + N_{t+1}}{3}$$

3.2 Visualization of growth dynamics

For the Sletkov-Arzamastsev model, experimental data were selected:

Time (iterations)	0	1	2	3	4
Number of people	10	25	60	130	200

For r=3.5 the sequence {xn} exhibits chaotic behavior (see Figure 1).

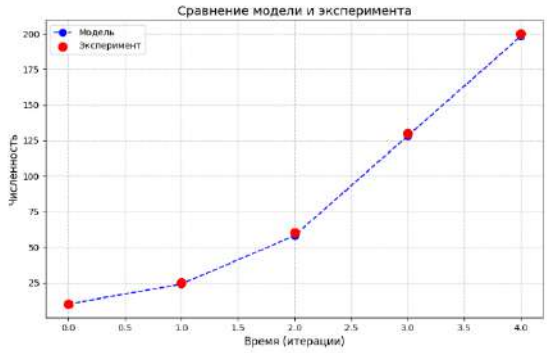

Figure 1: Graph of the dependence of the number of iterations on the number of population (ideal and experimental).

3.3 Numerical modeling

3.3.1. Solution of the difference equation

Analytical solution

For the logistic equation $x_{n+1} = r * x_n * (1 - x_n)$stable points:

$$x^* = 0 \text{ и } x^* = 1 - \frac{1}{r}$$

Stability:

For 1 < r < 3 – Stable non-trivial point;

For r > 3 – Bifurcations and chaos.

Numerical methods

An iterative approach is used to solve the equations:

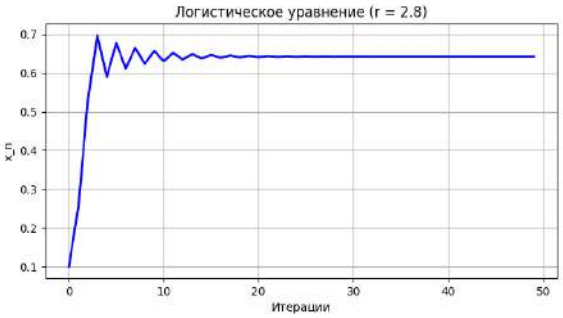

Figure 2: Iterative approach graph.

Sensitivity analysis to parameters

1. Effect of growth factor r:

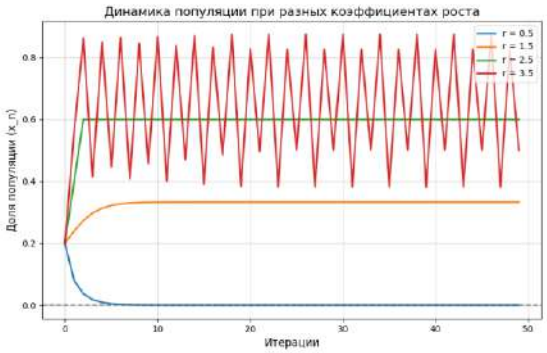

Figure 3: Graph of the system's dependence on the growth of the parameter r.

Simulation results

r value	**System behavior**	**Visualization**
r = 0.5	Rapid extinction	Monotonic decrease to 0
r = 1.5	Stabilization	Smooth exit to the plateau
r = 2.5	Oscillatory stabilization	Damped oscillations
r = 3.5	Chaotic mode	Non-periodic oscillations

Conclusions for r:

1. Threshold effect: When r=1, a qualitative change in the system's behavior occurs
2. Bifurcation: At r≈3.0 the system goes into oscillatory mode
3. Sensitivity: In the chaotic regime (r>3.5) small changes in r dramatically change the dynamics

Influence of diffusion coefficient D

For the spatial model:

$$N_{i,j}^{t+1} = N_{i,j}^t + r * N_{i,j}^t * \left(1 - \frac{N_{i,j}^t}{K}\right) + D * \nabla^2 N_{i,j}^t$$

Where:

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∇^2– discrete Laplacian (the difference between a cell and its neighbors)

D determines the rate of spread of the disease

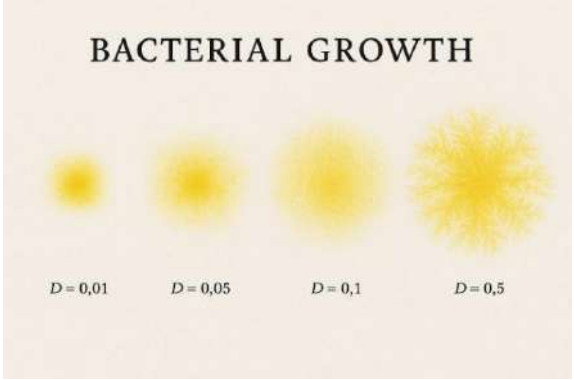

Figure 4: Demonstration of the dependence of the spread of the disease on the parameter D.

Simulation results

D value	**Nature of distribution**	**Visualization**
D = 0.01	Tight core, almost no blur.	A small area with sharp boundaries
D = 0.05	Light blur, colony begins to expand.	Weak "wings" are visible.
D = 0.1	Rapid spread	The Colony's Blurred Borders
D = 0.5	The disease has spread far and wide	Almost uniform distribution

Conclusions for D

1. Threshold effect: At D < 0.05 the colony practically does not expand
2. Colony form:
   - Small D: fractal-like structure
   - Large D: Smooth circular fronts
3. Speed of propagation: Linearly dependent on √D

Quantitative analysis

Let's add the calculation of the colony radius:

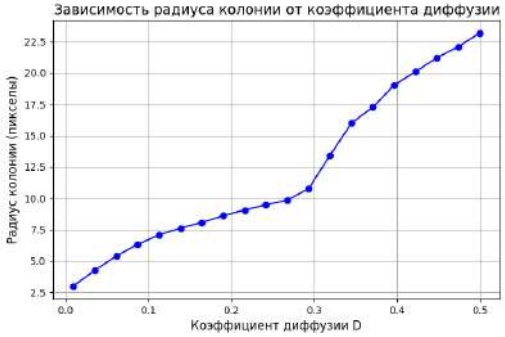

Figure 5: Dependence of the colony radius on the diffusion coefficient.

Comparison of the model with experimental data:

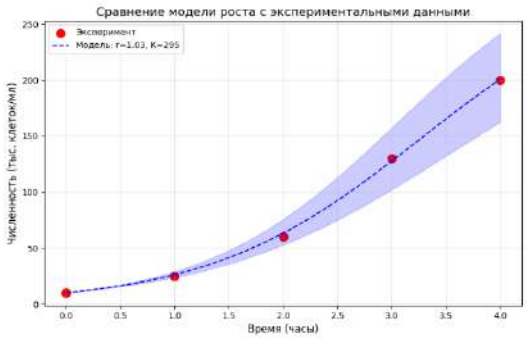

Figure 6: Visual comparison of simulated data with experimental data.

1. Simulation of disease control and treatment, where V_diseases > V_LMP and F_diseases > F_LMP. Given the parameters, when V_diseases > V_LMP and F_diseases > F_LMP, disease spread gains an advantage in two key factors: mobility and collision strength. This leads to rapid proliferation and disease dominance in the simulation, which is the expected outcome in these settings.

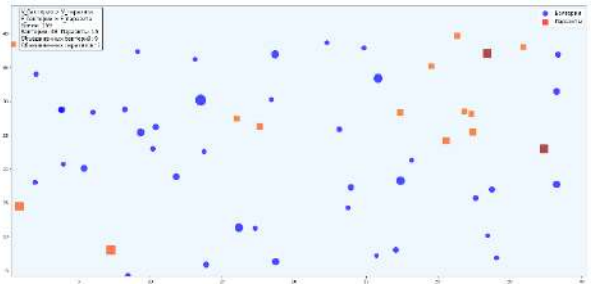

Figure 7: Simulation of the system after 150 iterations.

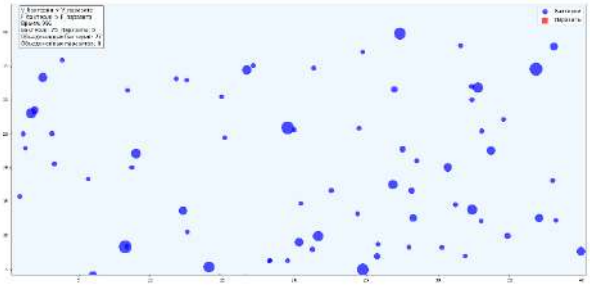

Figure 8: Simulation of the system after 1000 iterations.

2. With such parameters, when V_diseases < V_LPM and F_morbidity < F_treatment and preventive measures, the simulation inevitably leads to the dominance of therapy and prevention. They encounter diseases more often, win encounters and, thanks to the mechanism of increasing the size upon

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victory, accelerate their reproduction. This leads to the fact that LPM gradually capture the entire field, and the incidence disappears or ends up in an extremely small group

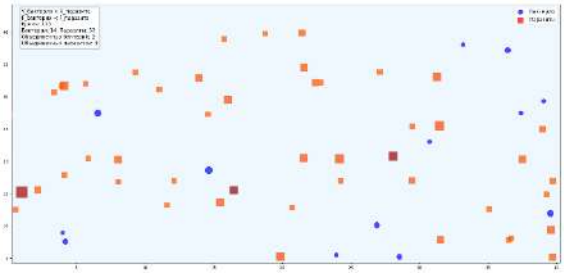

Figure 9: Simulation of the second situation after 120 iterations .

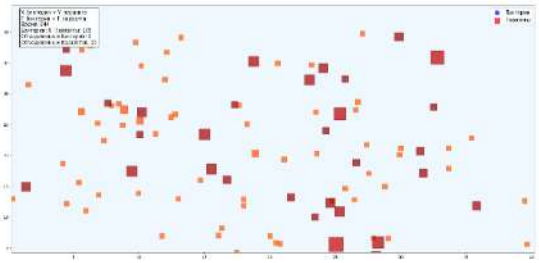

Figure 10: System state after 900 steps.

3. Given the parameters, when the LPMs have the same speed as the spread of the disease, and also have identical strength, the simulation leads to the fact that therapy and prevention easily pursue, attack and absorb the signs of the disease. The cumulative effect of increasing their size and energy, as well as more favorable conditions for reproduction, force the parasites to dominate the ecosystem. Thus, the final outcome of the simulation is complete (or almost complete) suppression of the spread of the disease in favor of treatment and preventive measures.

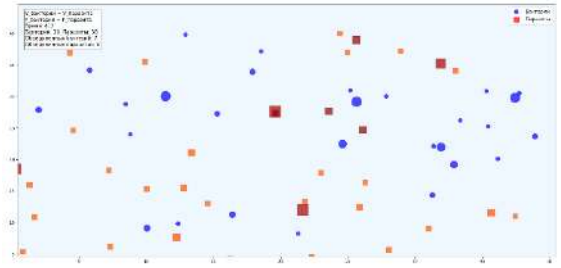

Figure 11: Simulation of the third situation at the moment of iteration 400.

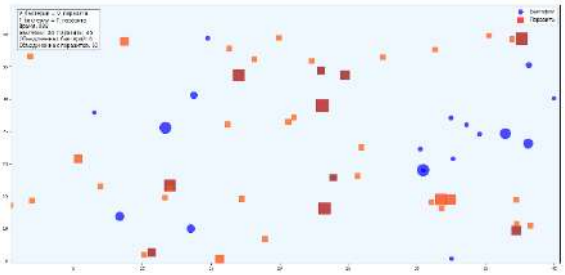

Figure 12: State of the situation after 900 “moves”.

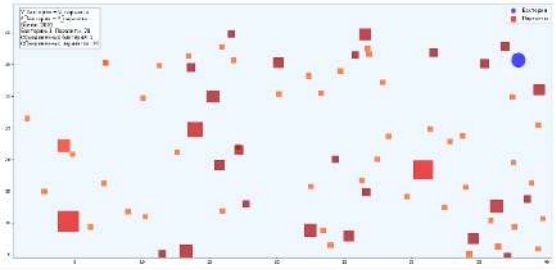

Figure 13: The final state of the system in the third experiment.

In general, the algorithm models the dynamic interaction of the spread of disease and treatment and preventive measures through:

- Random movement dependent on speed.
- Mechanisms of energy accumulation leading to reproduction.
- Probabilistic collisions where the winner gains extra size and energy and the loser is eliminated.
- Combining organisms to increase characteristics in clusters.

Thus, depending on the set parameters (speed and strength), the simulation shows which population gains an advantage and how these advantages allow one group to dominate the ecosystem. This algorithm allows for comparative analysis by changing the parameters to simulate different scenarios of struggle between organisms.

Graphs for simulated situations:

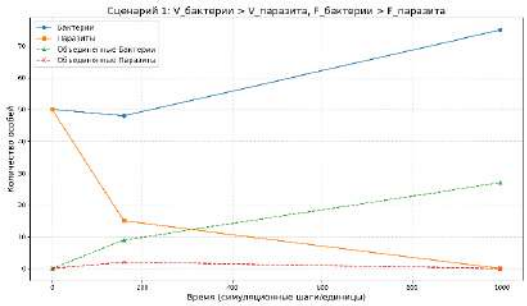

Figure 14: Graph of the dynamics of the first situation.

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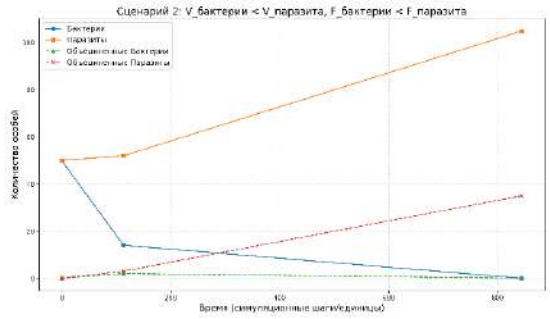

Figure 15: Generalized graph of modeling the second situation.

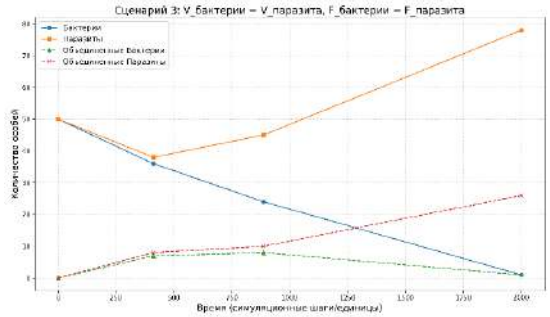

Figure 16: Simulation graph of situation No. 3.

4 CONCLUSION

Prevention and spread of diseases have a complex and close relationship. Preventive measures such as vaccination, hygiene and healthy lifestyle significantly reduce the risk of occurrence and spread of diseases, including infectious and non-infectious ones. The absence or insufficient prevention, on the contrary, can lead to an increase in the incidence and spread of infections.

Prevention is a key factor in combating the spread of diseases. Effective prevention can reduce morbidity, prevent severe complications, protect the population from mass epidemics and improve overall health..

The work done laid the foundation for the analysis of complex biological systems using mathematical modeling methods. Further research will expand the applicability of the model in biotechnology, medicine and ecology, ensuring more accurate forecasting and management of the growth of morbidity by organizing treatment and preventive measures.

For example, in medicine, the proposed model can be used to study the dynamics of infection development and develop new approaches to antimicrobial therapy. Analysis of the interaction of microorganisms with the immune system, as well as forecasting the effectiveness of various drugs will become possible thanks to improved mathematical methods.

In the field of ecology, the proposed model can be applied to assess the impact of various factors, such as environmental pollution, on the growth and development of microorganisms in natural ecosystems. This will allow more accurate prediction of the consequences of anthropogenic impact and the development of effective strategies for environmental protection.

Thus, further development and improvement of the proposed model has enormous potential for solving a wide range of problems in various fields of science and technology.

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