DOI: 10.63550/ICEIP.2025.79.45.036

Optimizing Large-Scale Database Search with Grover's Quantum Algorithm

Sh.A.Toirov[1][1], X.U.Akbarov[2][2], O.X.Turakulov[1][3],X.A.Nazarov[1][4] and R.A. Yusupov[1][5]

[1]*Tashkent Institute of Management and Economics, 114, Shota Rustaveli, Tashkent, 100000, Uzbekistan*

[2]*Samarkand Agroinnovations and Research University, Dakhbed town, A.Temur street, 7, Samarkand, 141001, Uzbekistan*

tashuxrat@mail.ru, akbarovhusan@gmail.com, o_xolmirzayevich@mail.ru, Xolmumin.nazarov1672@gmail.com, neouzbek77@gmail.com

Keywords: the quantum algorithm, Grover's method, global optimization, intellectual management.

Abstract: The study examines the use of Grover's quantum algorithm to optimize global search problems, demonstrating its quadratic speedup over classical methods for improved efficiency. The research tackles practical challenges, including quantum noise, scalability, and resource constraints, by proposing innovative solutions. The techniques developed integrate quantum search principles into large-scale optimization tasks across diverse fields. This work lays a foundational framework for applying quantum algorithms to solve complex scientific and industrial problems.

1 INTRODUCTION

It is becoming more crucial in the world to learn quantum algorithm, and to improvise, develop and introduce the methods and algorithms of solving problems by these algorithms. Nowadays, the tasks solved by quantum algorithms are generating more efficient results than the problems that are solved using other algorithms (Goldberg, 2000). At present, special attention is paid to the analytical analysis of mathematical models of these algorithms and the creation of quantum computers that run on the basis of quantum algorithms. In quantum computing (quantum algorithms) the quality (feature) of the process that is being studied is determined as a result of direct parallel calculations (Lahoz-Beltra, 2004). In addition, the solution of the problems that are difficult to solve or for the ones that are not possible to be solved algorithmically by traditional (classic) methods.

Although Grover's algorithm is generally considered useful for database searching, the basic ideas underlying this algorithm are applicable in a much broader context. This approach is used to speed up search algorithms that can build a "quantum oracle" that separates the needle from the haystack. It uses the term ancilla (Ancilli are extra bits used to achieve specific computational goals (such as in reversible computation)) or auxiliary qubit (ancilla qubit) to refer to some additional qubits used by an algorithm.

2 MAIN BODY

This algorithm searches through $N = 2^n$ an unordered set of elements in order to find the element that satisfies some conditions. Currently, while the best classical search algorithm on unstructured data takes time $O(N)$, Grover's algorithm allows for a

[1] https://orcid.org/0000-0002-6709-0480
[2] https://orcid.org/0000-0003-1387-0892
[3] https://orcid.org/0009-0007-7676-3476
[4] https://orcid.org/0009-0009-9732-4952
[5] https://orcid.org/0000-0003-4025-4198

DOI: 10.63550/ICEIP.2025.79.45.036

quadratic speedup of search on a quantum computer in operations just $O\left(\sqrt{N}\right)$.

Grover's search algorithm is considered to be one of the best methods of quantum algorithms, and it shows that when the classical algorithms of quantum system is used, it depends on the slowness of operation time, and that it can be used in order to improve its quality. In this case, in order to reach a high speed, Grover's algorithm bases on the quantum super-position of the processes (Han, Kim, 2002). As numerous quantum algorithms, Grover's algorithm also sets off by putting n qubit registers of the machine into the super-position that is equal to all the possible 2^ncases. It is important to remember that the amplitude associated with each possible configuration of each qubit is equal to $\frac{1}{\sqrt{2^n}}$ and the probability of being of the 2^n state of the system in any state is equal to $\frac{1}{2^n}$. All these possible states correspond to all possible entries in the database of Grover's algorithm, and therefore, starting with a given amplitude assigned to each element in the search space, each element is considered simultaneously in quantum superposition and amplitudes are controlled from there (Lahoz-Beltra, 2015).

Along with the superposition of states, Grover's algorithm belongs, in general, to the family of quantum algorithms that use amplitude amplifiers, which take the advantage of quantum amplitudes that distinguish amplitudes from probabilities. The key to these algorithms is a selective displacement of one state of quantum system, of its space that satisfy some kind of condition in each iteration. These amplitude amplifier algorithms are so unique to quantum calculating that such feature of amplitudes has no parallel in classical probabilities.

3 METHODS

Grover's algorithm utilizes a quantum register composed of n - qubits, where n is the number of qubits required to represent a search space of size $N = 2^n$. Initially, all qubits are set to the ground state $|0\rangle$:

$$|0\rangle^{\otimes n} = |0\rangle \quad (1)$$

After a sufficient number of iterations of Grover's iteration are accomplished, a classical measurement is performed to determine the result, this completion of the algorithm continues until the probability $O(1)$.

The steps of Grover's algorithm are implemented and summarized as the following:

Input:

- $O|x\rangle = (-1)^{f(x)}|x\rangle$ is quantum oracle O, which performs the operation, where $f(x) = 0$ is $f(x_0) = 1$ for all the $0 \le x < 2^n$, except for $x \ne x_0$.
- A qubit $|0\rangle$ initiated to state n
- Output: x_0

The running time: the operations $O(\sqrt{2^n})$, with the probability $O(1)$.

Process:

1. The initial state $|0\rangle^{\otimes n}$
2. Using Hadamard conversion for all the qubits

$$H^{\otimes n}|0\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}}\sum_{x=0}^{2^n-1}|x\rangle = |\psi\rangle$$

3. Using Grover's iteration $R \approx \frac{\pi}{4}\sqrt{2^n}$ times

$$[2|\psi\rangle\langle\psi| - I]^R|\psi\rangle \approx |x_0\rangle$$

4. Measuring the register x_0

4 RESULTS

Mathematical solutions of the above-mentioned information through a specific example are as follows. Let's say the expression of the function is $f(x) = \frac{x+6}{2+cos(x)}$ and the oracle accepts values between 0 and 32. The next step is to consider the case where $N = 32 = 2^5$ is equal, and the desired state x_0 is represented by a string of 11110 bits.

To describe this process, $n = 5$ consists of qubits, i.e.

$$|x\rangle = \alpha_0|00000\rangle + \alpha_0|00001\rangle + ... + \alpha_{31}|11111\rangle$$

where a_i - $|i\rangle$ is the amplitude of the state. Grover's algorithm starts from system 0

$$1|00000\rangle$$

and then the Hadamard transform is applied to obtain an equal amplitude associated with each state $\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{32}} = \frac{1}{4\sqrt{2}}$ so that the solution to the

DOI: 10.63550/ICEIP.2025.79.45.036

problem is equal to the probability of being in one of the 32 possible states.

$$H^5|00000\rangle = \frac{1}{4\sqrt{2}}|00000\rangle + \frac{1}{4\sqrt{2}}|00001\rangle + ... + \frac{1}{4\sqrt{2}}|11111\rangle = \frac{1}{4\sqrt{2}}\sum_{x=0}^{31}|x\rangle = \psi$$

Three Grover iterations are sufficient to solve the problem, i.e. $\frac{\pi}{4}\sqrt{N} = \frac{\pi}{4}\sqrt{32} = \sqrt{2}\pi \approx 4,4$, which turns up to 4 iterations.

At each iteration, the quantum oracle *O* must first be invoked, followed by an inversion by averaging or diffusion transformation. The oracle query negates the condition amplitude $|x_0\rangle$ in which case $|11110\rangle$ gives the configuration.

$$|x\rangle = \frac{1}{4\sqrt{2}}|00000\rangle + \frac{1}{4\sqrt{2}}|00001\rangle + ... - \frac{1}{4\sqrt{2}}|11110\rangle + \frac{1}{4\sqrt{2}}|11111\rangle$$

In the next case, a diffusion transformation $2|\psi\rangle\langle\psi| - I$ is performed, which increases the amplitudes from the mean value, decreases it if the difference is negative

$$\left[2|\psi\rangle\langle\psi| - I\right]\left[|\psi\rangle - \frac{2}{4\sqrt{2}}\right] = 2|\psi\rangle\langle\psi|\psi\rangle - |\psi\rangle - 2\left(\frac{2}{4\sqrt{2}}\right)|\psi\rangle\langle\psi|11110\rangle + \frac{1}{2\sqrt{2}}|11110\rangle =$$

$$= 2|\psi\rangle - |\psi\rangle - \frac{1}{8}|\psi\rangle + \frac{1}{2\sqrt{2}}|11110\rangle = \frac{7}{8}|\psi\rangle + \frac{1}{2\sqrt{2}}|11110\rangle =$$

$$= \frac{7}{8}\left[\frac{1}{4\sqrt{2}}\sum_{\substack{x=0\\x\neq x_0}}^{31}|x\rangle + \frac{1}{4\sqrt{2}}|11110\rangle\right] + \frac{1}{2\sqrt{2}}|11110\rangle = \frac{7}{32\sqrt{2}}\sum_{\substack{x=0\\x\neq x_0}}^{31}|x\rangle + \frac{7}{32\sqrt{2}}|11110\rangle + \frac{1}{2\sqrt{2}}|11110\rangle =$$

$$= \frac{7}{32\sqrt{2}}\sum_{x=0}^{31}|x\rangle + \frac{23}{32\sqrt{2}}|11110\rangle$$

Now the $|x\rangle$ used above will be:

$$|x\rangle = \frac{7}{32\sqrt{2}}|00000\rangle + \frac{7}{32\sqrt{2}}|00001\rangle + ... + \frac{23}{32\sqrt{2}}|11110\rangle + \frac{7}{32\sqrt{2}}|11111\rangle$$

This completes the first iteration. We apply the same two changes in the second iteration.

$$|x\rangle = \frac{7}{32\sqrt{2}}|00000\rangle + \frac{7}{32\sqrt{2}}|00001\rangle + ... - \frac{23}{32\sqrt{2}}|11110\rangle + \frac{7}{32\sqrt{2}}|11111\rangle =$$

$$= \frac{7}{32\sqrt{2}}\sum_{\substack{x=0\\x=x_0}}^{31}|x\rangle - \frac{23}{32\sqrt{2}}|11110\rangle = \frac{7}{32\sqrt{2}}\sum_{\substack{x=0\\x=x_0}}^{31}|x\rangle - \frac{7}{32\sqrt{2}}|11110\rangle - \frac{23}{32\sqrt{2}}|11110\rangle =$$

$$= \frac{7}{32\sqrt{2}}\sum_{\substack{x=0\\x=x_0}}^{31}|x\rangle - \frac{30}{32\sqrt{2}}|11110\rangle = \frac{7}{8}|\psi\rangle - \frac{15}{16\sqrt{2}}|11110\rangle$$

After querying Oracle and applying diffusion transformation.

DOI: 10.63550/ICEIP.2025.79.45.036

$$\left[2|\psi\rangle\langle\psi|-I\right]\left[\frac{7}{8}|\psi\rangle-\frac{15}{16}|11110\rangle\right]=2\left(\frac{7}{8}\right)|\psi\rangle\langle\psi|\psi\rangle-\frac{7}{8}|\psi\rangle-2\left(\frac{15}{16\sqrt{2}}\right)|\psi\rangle\langle\psi|11110\rangle+$$

$$+\frac{15}{16}|11110\rangle=\frac{7}{4}|\psi\rangle-\frac{7}{8}|\psi\rangle-\frac{15}{64}|\psi\rangle+\frac{15}{16\sqrt{2}}|11110\rangle=\frac{41}{64}|\psi\rangle+\frac{15}{16\sqrt{2}}|11110\rangle=$$

$$=\frac{41}{64}\left[\frac{1}{4\sqrt{2}}\sum_{\substack{x=0\\x=x_0}}^{31}|x\rangle+\frac{1}{4\sqrt{2}}|11110\rangle\right]+\frac{15}{16\sqrt{2}}|11110\rangle=$$

$$=\frac{41}{256\sqrt{2}}\sum_{x=0}^{31}|x\rangle+\frac{41}{256\sqrt{2}}|11110\rangle+\frac{15}{16\sqrt{2}}|11110\rangle=\frac{41}{256\sqrt{2}}\sum_{x=0}^{31}|x\rangle+\frac{281}{256\sqrt{2}}|11110\rangle$$

Nov the $|x\rangle$ used above will be:

$$|x\rangle=\frac{41}{256\sqrt{2}}|00000\rangle+\frac{41}{256\sqrt{2}}|00001\rangle+.....+\frac{281}{256\sqrt{2}}|11110\rangle+\frac{41}{256\sqrt{2}}|11111\rangle$$

This second one completes the iteration. We apply the same two changes in the third iteration.

$$|x\rangle=\frac{41}{256\sqrt{2}}|00000\rangle+\frac{41}{256\sqrt{2}}|00001\rangle+.....-\frac{281}{256\sqrt{2}}|11110\rangle+\frac{41}{256\sqrt{2}}|11111\rangle=$$

$$=\frac{41}{256\sqrt{2}}\sum_{\substack{x=0\\x\neq x_0}}^{31}|x\rangle-\frac{41}{256\sqrt{2}}|11110\rangle-\frac{281}{256\sqrt{2}}|11110\rangle=\frac{41}{256\sqrt{2}}\sum_{x=0}^{31}|x\rangle-\frac{322}{256\sqrt{2}}|11110\rangle=$$

$$=\frac{41}{64}|\psi\rangle-\frac{161}{128\sqrt{2}}|11110\rangle$$

After querying Oracle and applying diffusion transformation.

$$\left[2|\psi\rangle\langle\psi|-I\right]\left[\frac{41}{64}|\psi\rangle-\frac{161}{128\sqrt{2}}|11110\rangle\right]=2\left(\frac{41}{64}\right)|\psi\rangle\langle\psi|\psi\rangle-\frac{41}{64}|\psi\rangle-2\left(\frac{161}{168\sqrt{2}}\right)|\psi\rangle\langle\psi|11110\rangle=$$

$$=\frac{41}{32}|\psi\rangle-\frac{41}{64}|\psi\rangle-\frac{161}{512\sqrt{2}}|\psi\rangle+\frac{161}{128\sqrt{2}}|11110\rangle=\frac{167}{512\sqrt{2}}|\psi\rangle+\frac{161}{128\sqrt{2}}|11110\rangle=$$

$$=\frac{167}{512\sqrt{2}}\left[\frac{1}{4\sqrt{2}}\sum_{\substack{x=0\\x\neq x_0}}^{31}|x\rangle+\frac{1}{4\sqrt{2}}|11110\rangle\right]+\frac{161}{128\sqrt{2}}|11110\rangle=\frac{167}{2048\sqrt{2}}\sum_{\substack{x=0\\x\neq x_0}}^{31}|x\rangle+\frac{2743}{2048\sqrt{2}}|11110\rangle$$

Now, when the process is observed, the correct solution state $|11110\rangle$ has the measured probability $\frac{2743}{2048\sqrt{2}}\approx 0,95\%$ and the wrong probability state $\frac{167}{2048\sqrt{2}}\approx 0,05\%$.

The above mathematical solutions are based on the results obtained after executing the program on a classical computer using a quantum algorithm. First, the program creates a superposition state.

[[0.1767767]
[0.1767767]
[0.1767767]
...
[0.1767767]
[0.1767767]
[0.1767767]]

Second, the oracle O determines the maximum $|\psi\rangle$

And so, $O|\psi\rangle^{Q(t)}=(-1)^{f(x)}|\psi\rangle^{Q(t)}$ when applied, the following superposition condition is obtained:

[[0.1767767]
[0.1767767]
...
[0.1767767]
[-0.1767767]
[0.1767767]
[0.1767767]]

This second step indicates the number of repetitions given. Grover's maximum number of repetitions is calculated as follows:

$$\frac{\pi}{4}\sqrt{2^n}$$

DOI: 10.63550/ICEIP.2025.79.45.036

n the number of qubits or the length of the quantum chromosome, so $n=5$ in the example of the function described in the problem.

As a result of repeating the second step

[[0.15467961]
...
[0.15467961]
[-0.508233]
[0.15467961]
[0.15467961]
[0.15467961]]

In the third step, the oracle O determines the maximuml $|\psi\rangle$ and the following results are obtained as a result of iteration.

Fourth and last, Grover's diffusion operator locates the chromosome with the specified state at $|\psi\rangle^{Q(t)}$. Therefore, the following process $|\psi\rangle^{Q(t)} = G|\psi\rangle^{Q(t)}$ execution results.

[[0.11324757]
[0.11324757]
...
[0.11324757]
[-0.77616018]
[0.11324757]
[0.11324757]
[0.11324757]]

In the next step, we get the result

[[0.05765959]
[0.05765959]
...
[0.94706733]
[0.05765959]
[0.05765959]
[0.05765959]]

Finally, $|\psi\rangle^{Q(t)}$ when done, the state pointed to by the maximum chromosome is obtained.

That is, the state of the quantum algorithm $|30\rangle$, chromosome $|11110\rangle$ and the objective function $|1671\rangle$ will be equal to (Toirov, Mingboyev, Parmonov, Kudratov, 2024).

5 DISCUSSION

As you can see from Figure 1 in this article, solving the problem leads to four iterations of Grover's iteration, i.e., $\frac{\pi}{4}\sqrt{N} = \frac{\pi}{4}\sqrt{32} = \sqrt{2}\pi \approx 4,4$, which goes up to 4 iterations. At each iteration, it first uses the quantum oracle O, and then performs an inversion on the average or diffusion transform. It is clear from the process of solving the problem mathematically that Grover's algorithm makes it easier to reach the solution by increasing the amplitude. That is, in our problem, the optimal solution of the given function is considered to be 0.94.

6 FUTURE WORK AND RESEARCH CHALLENGE

In this article, the main goal of obtaining results is the process of solving the problem of global optimization through algorithms, and the extent to which the current information about the subject of research is modeled of the method of solving the problem of global optimization through quantum algorithms it was considered that it is important to ensure that it is used for the intended purpose, that is, the adequacy of the model. The used quantum algorithm is distinguished by the fact that it solves the optimization problem faster than classical computers, and we achieved the result set before us. The obtained results proved to be the solution to the problem. We emphasize that the algorithm proposed here can be easily implemented in near-future devices.

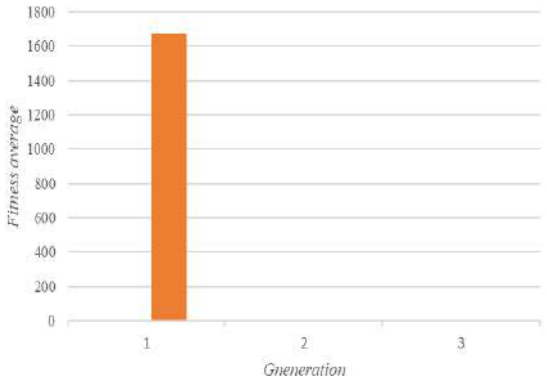

Figure 1: The graph of the result of the optimization carried out.

7 CONCLUSION

The problems for which quantum algorithms can be potentially used:

a) effective calculating the number of the solutions to given search problem;

b) speeding up NP problems;

c) speeding up the search of keys in cryptosystems;

d) finding the shortest way;

e) searching or analyzing necessary data in unordered databases.

Grover's algorithm is more likely to get a correct answer than a wrong answer, when the input size is $N = 2^n$, the error only decreases with the input size. Even though Grover's algorithm is probabilistic, the error gets negligible as N gets larger.

REFERENCES

Goldberg, D.E., 2000. *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley: Reading, MA, USA, pp. 1-432.

Lahoz-Beltra, R., 2004. *Bioinformatica: Simulacion, Vida Artificial e Inteligencia Artificial*. Ediciones Diaz de Santos: A Coruna, Spain, pp. 237-323.

Perales-Gravan, C., Lahoz-Beltra, R., 2008. An AM radioreceiver designed with a genetic algorithm based on a bacterial conjugation genetic operator. *IEEE Trans. Evolut. Comput.* 12, pp.129-142.

Ribeiro Filho, J.L., Treleaven, P.C., Alippi, C., 1994. Genetic-algorithm programming environments. *IEEE Comput. 24*, pp. 28-43.

Feynman, R.P., 2001. Simulating physics with computers. *Int. J. Theor. Phys.* 21, 467-488.

Fraser, A.S., 1990. Simulation of genetic systems by automatic digital computers. *Aust. J. Biol. Sci.* 10,484^91.

Han, K.-H., Kim, J.-H., 2002. Quantum-inspired evolutionary algorithm for a class of combinatorial optimization. *IEEE Trans. Evolut. Comput.* 6, pp. 580-593.

Ying, M., 2010. Quantum computation, quantum theory and AI. *Artif. Intell.* 174,162-176.

Lahoz-Beltra, R., 2015. University of Stirling, Computing Science and Mathematics School of Natural Sciences, Stirling, Scotland, United Kingdom. Quantum Genetic Algorithms for Computer Scientists. Computing Science Seminars, Spring 2015, 26 June. Personal communication.

Susskind, L., Friedman, A., 2015. *Quantum Mechanics: The Theoretical Minimum;* Penguin Books: London, UK. pp. 1-364.

Boghosian, B.M., Taylor, W., 2002. Simulating quantum mechanics on a quantum computer. *Phys. D* 120, pp. 30-42.

Alfonseca, M., Ortega, A., de La Cruz, M., Hameroff, S.R., Lahoz-Beltra, R., 2015. A model of quantum-von Neumann hybrid cellular automata: Principles and simulation of quantum coherent superposition and decoherence in cytoskeletal microtubules. *Quantum Inf. Comput. 15*, pp. 22-36.

Zeiter, D., 2008. A Graphical Development Environment for Quantum Algorithms. Master's Thesis, Department of Computer Science, ETH Zurich, Zurich, Switzerland.

QCAD: GUI Environment for Quantum Computer Simulator. Available online: http://qcad.osdn.jp/ (accessed on 25 April 2016).

jQuantum-Quantum Computer Simulation Applet. Available online: http://jquantum.sourceforge.net/ jQuantumApplet.html (accessed on 25 April 2016).

Hayes, B., 2014. Programming your quantum computer. *Am. Sci. 102*, pp.22-25.

QCL-A Programming Language for Quantum Computers. Available online: http://tph.tuwien.ac.at/ ~oemer/qcl.html (accessed on 25 April 2016).

Al Daoud, E., 2007. Quantum computing for solving a system of nonlinear equations over GF(q). *Int. Arab J. Inf. Technol. 4*, pp.201-205.

Lahoz-Beltra, R., 2016. Simple genetic algorithm (SGA). Figshare.

Lahoz-Beltra, R., Perales-Gravan, C., 2010. A survey of nonparametric tests for the statistical analysis of evolutionary computational experiments. Int. J. Inf. Theor. Appl. 17, pp.49–61.

Toirov, Sh., Narmuradov, U., 2021. "Processes of applying of quantum genetic algorithm in function optimization." *AIP Conference Proceedings*. Vol. 2365. No. 1. AIP Publishing.

Toirov, Sh. A., 2022. "Optimization with Quantum Algorithm that is Based on Grover's Method." *International Scientific and Practical Conference on Information Technologies and Intelligent Decision Making Systems*. Cham: Springer Nature Switzerland.

Toirov, Sh., 2023. "DESIGN OF GROVER'S QUANTUM SEARCH ALGORITHM FOR 2-KBIT STATES." *Universum: технические науки* 12-7 (117): 8-11.

Toirov, Sh. A., Mingboyev, U. X., Parmonov, A. A., Kudratov, R. B., 2024. "Designing Grover's quantum searching algorithm for 2-qubit and 3-qubit states of quantum systems" *III International Scientific and Practical Conference "Technologies, Materials Science and Engineering", AIP Conf. Proc. 3243, 010001*.